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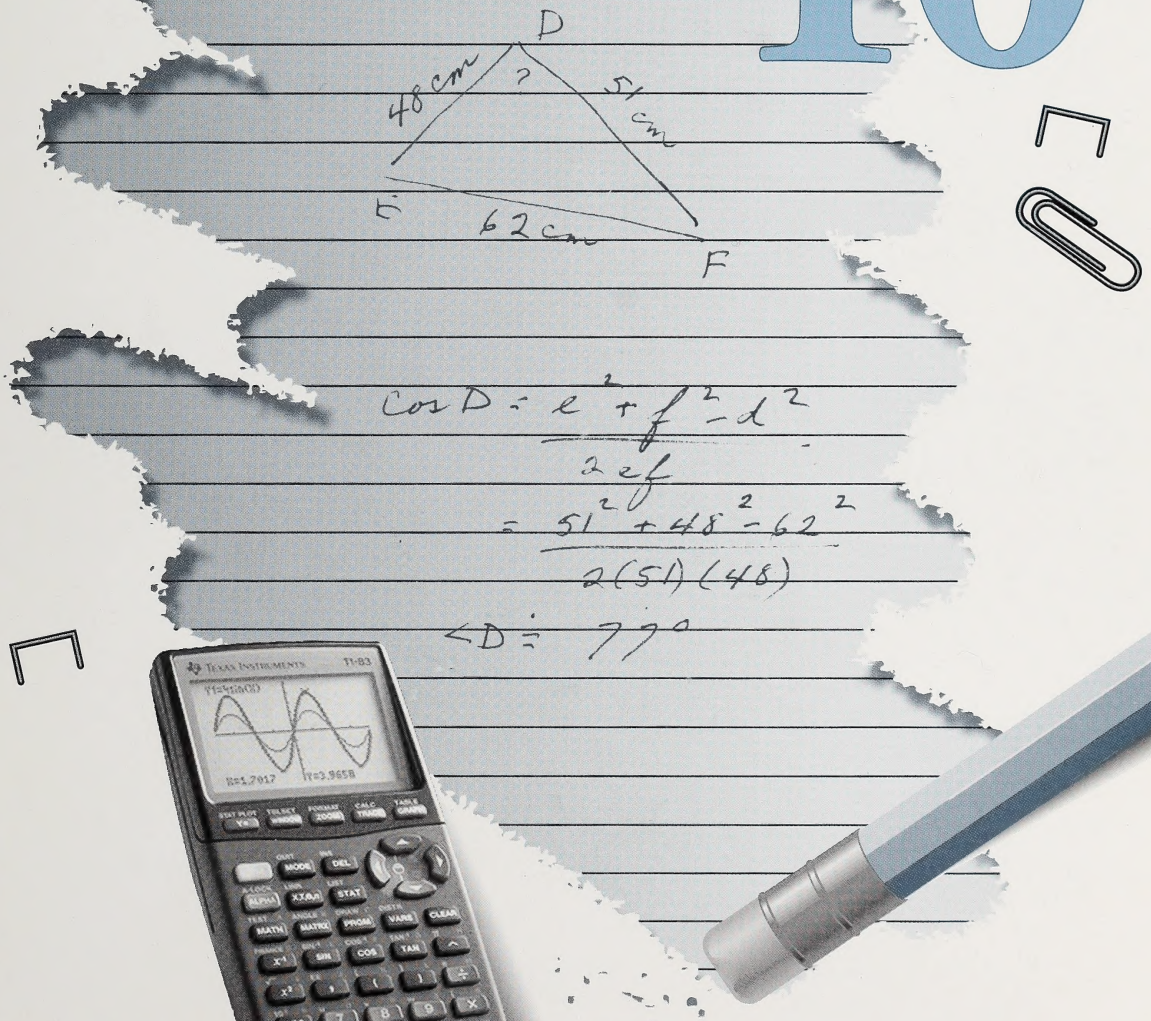
Module 3

MATHEMATICS 10

Algebra



Learning Technologies Branch

Alberta LEARNING

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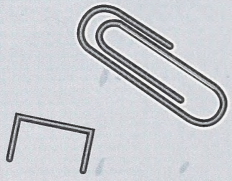
Pure MATHEMATICS 10

Module 3
Algebra



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Pure Mathematics 10
Module 3: Algebra
Student Module Booklet
Learning Technologies Branch
ISBN 0-7741-2142-4

This document is intended for	
Students	✓
Teachers	✓
Administrators	
Home Instructors	
General Public	
Other	

You may find the following Internet sites useful:

- Alberta Learning, **http://www.learning.gov.ab.ca**
- Learning Technologies Branch, **http://www.learning.gov.ab.ca/ltb**
- Learning Resources Centre, **http://www.lrc.learning.gov.ab.ca**

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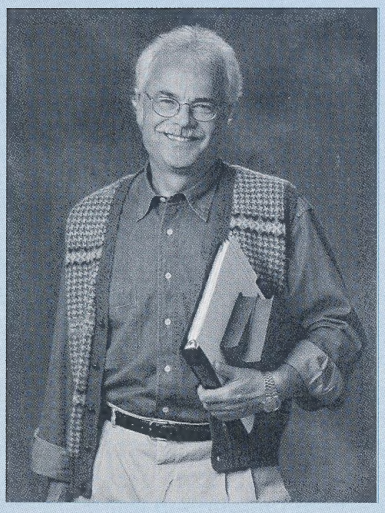
Welcome



Welcome to Module 3. We hope you'll enjoy your study of Algebra.

Pure Mathematics 10

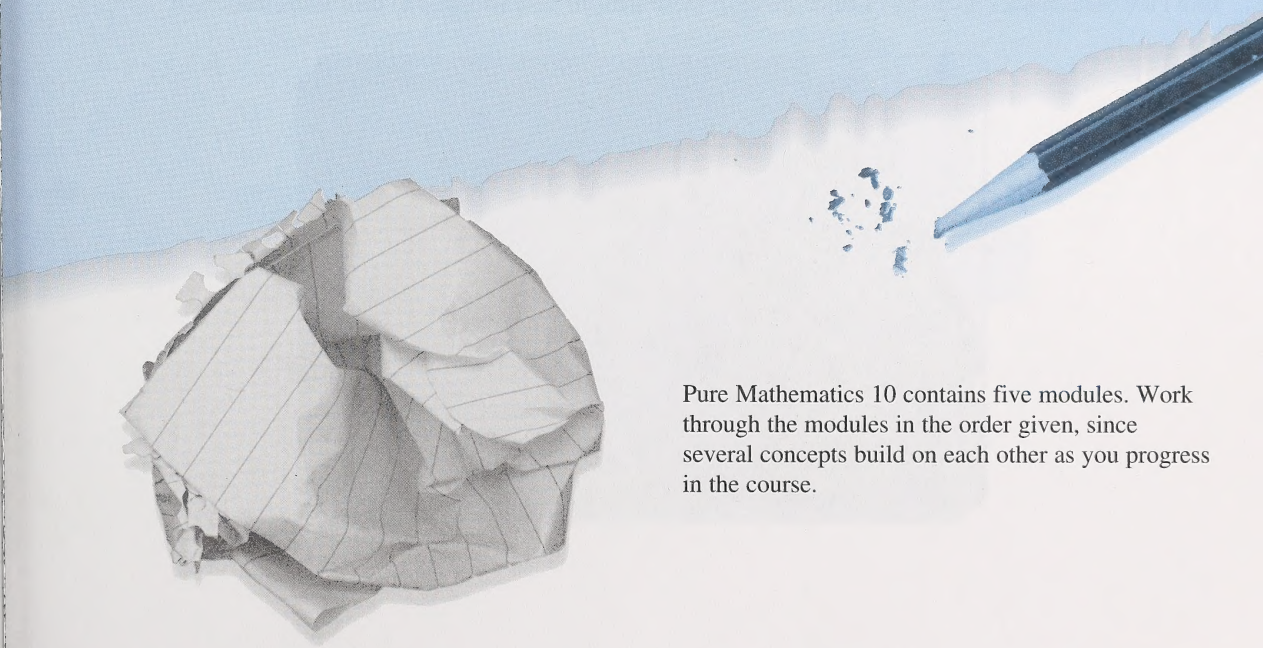
Module 1
Statistics and Data Tables

Module 2
Patterns and Relations

Module 3
Algebra

Module 4
Number Connections

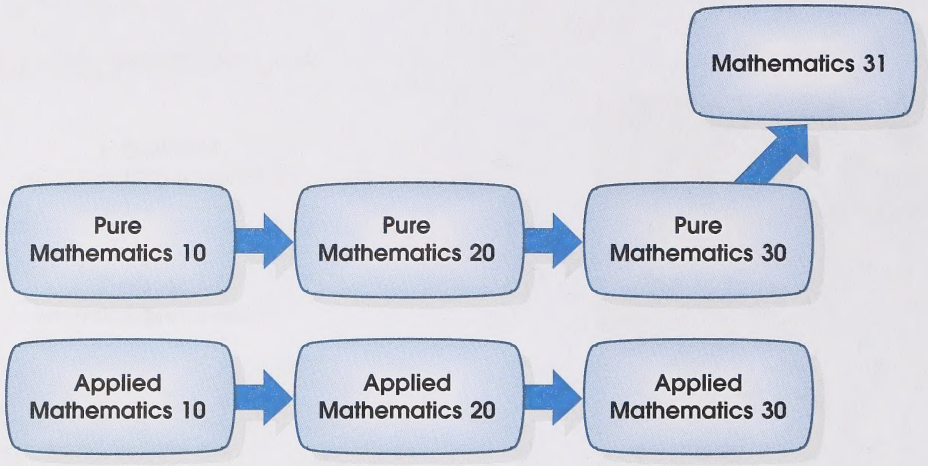
Module 5
Trigonometry



Pure Mathematics 10 contains five modules. Work through the modules in the order given, since several concepts build on each other as you progress in the course.

High School Mathematics Courses

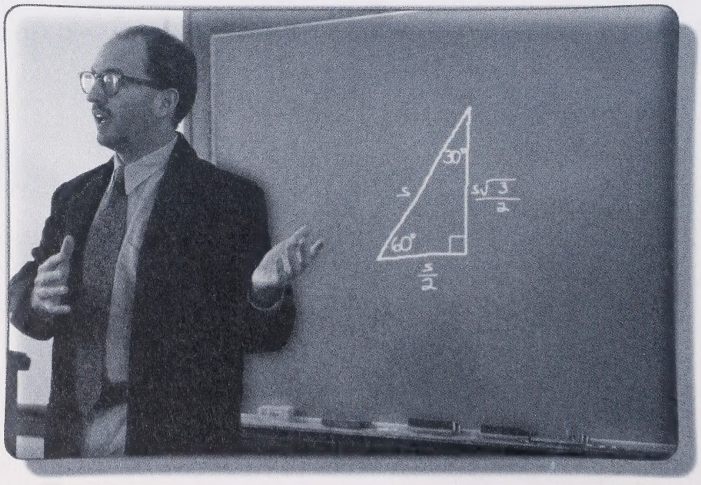
Pure Mathematics 10 is the first course in the Pure Mathematics 10–20–30 sequence of courses. Many students who take the Pure Mathematics 10–20–30 will also choose to take Mathematics 31. Another sequence of courses is Applied Mathematics 10–20–30.



How Do the Sequences Differ?

Each sequence of courses is designed for students with different mathematical strengths and interests. The Pure Mathematics 10–20–30 sequence is intended for students who are strong in mathematical theory and algebra. The Applied Mathematics 10–20–30 sequence is better suited to students who prefer to solve mathematical problems using numerical reasoning or geometry.

Each sequence of courses is designed for students with different career plans. Students who plan to take courses such as commerce, engineering, physics, and engineering technologies in post-secondary education will need to take Pure Mathematics 30 in high school. Some of these students will also need to take Mathematics 31.

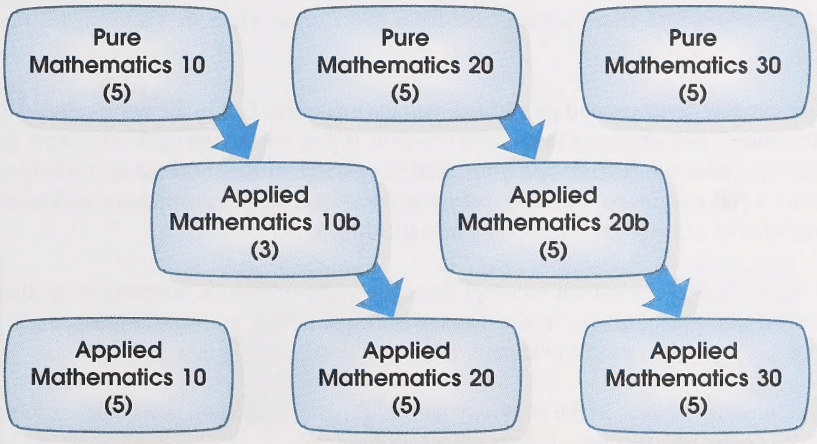


Can I Transfer to the Applied Mathematics Sequence of Courses?

You should be aware that the applied and pure mathematics courses do have some topics in common; other topics are independent.

Applied Topics	Common Topics	Pure Topics
• linear programming • data tables and trends • design and layout • metric and imperial measure • data presentation • vectors and matrices • periodic, fractal, and recursive patterns • financial decision making • costing and design problems	• spreadsheets • line segments and linear graphs • scaling • triangles • surveys • financial mathematics • quadratic functions • circle geometry • the bell curve	• irrational numbers • exponents • polynomial and rational expressions • mathematical expectation • growth patterns • linear and non-linear systems • operations on functions • mathematical reasoning • exponential and logarithmic functions • conics • combinations • trigonometric functions

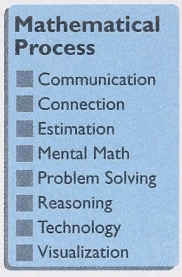
If you want to transfer from the Pure Mathematics 10–20–30 sequence to the Applied Mathematics 10–20–30 sequence at a future time, you won't have to repeat the topics that are common to pure mathematics and applied mathematics. If you decide to transfer to the Applied Mathematics sequence of courses after successfully completing Pure Mathematics 10, you may take the 3-credit course called Applied Mathematics 10b or you may take the 5-credit course called Applied Mathematics 10. If you decide to transfer after successfully completing Pure Mathematics 20, you will take the 5-credit course called Applied Mathematics 20b. The two bridging courses are shown in the following diagram:


Pure Mathematics 10 (5)
Pure Mathematics 20 (5)
Pure Mathematics 30 (5)
Applied Mathematics 10b (3)
Applied Mathematics 20b (5)
Applied Mathematics 10 (5)
Applied Mathematics 20 (5)
Applied Mathematics 30 (5)

Mathematical Processes

Throughout the course you will be expected to do the following:

- communicate mathematically
- connect mathematical ideas to everyday experiences and to concepts in other disciplines
- develop and use estimation strategies
- develop and use mental math strategies
- develop and use problem-solving strategies
- reason and justify your thinking
- select and use appropriate technologies as tools to solve problems
- use visualization to assist in processing information, making connections, and solving problems



This icon shows which mathematical process you are expected to perform.

In order to develop these mathematical processes more fully, you are encouraged to ask someone who is also taking Pure Mathematics 10 to be your study partner. You will find that having a friend to discuss mathematics with will make your studying more enjoyable.

You are also encouraged to record your mathematical ideas in a journal. Here are some suggestions for organizing your journal:

- Keep a section on new concepts and procedures you have learned. Get in the habit of describing new concepts, procedures, and strategies in your own words. If you are having difficulty, write your questions in your journal before discussing them with your teacher or study partner. Record useful ways to help you remember what a concept means. Make graphic organizers (such as context webs, Venn diagrams, or organizational charts) to help you connect mathematical ideas.

- Keep a section on your mathematical accomplishments. This can include solutions to problems that you are proud of solving. It can also include landmark events, such as when you finally grasped a difficult concept (an "aha!") or when you first used a calculator or spreadsheet program in a new way.

- Keep a section on mathematics in the everyday world. This could include newspaper articles, cartoons, and descriptions of careers that require a mathematical background.

Resources You Will Need

In order to complete Pure Mathematics 10 successfully, you will need to have access to the following resources, which can be purchased through the Learning Resources Centre:

- the *MATHPOWER™ 10* textbook and data disks, Western Edition, published by McGraw-Hill Ryerson (1998)

- a graphing calculator
  Any of the following graphing calculators may be used: *Texas Instruments* TI-83, TI-83 Plus, TI-86, TI-89, TI-92, or TI-92 Plus; *Casio* Algebra FX-2.0, CFX-9850 GA-Plus, CFX-9850 G, CFX-9800 G, or FX-9700 series; *Sharp* EL-9600C, EL-9600, EL-9200, or EL-9300; or *Hewlett-Packard* HP 39g (only approved for 2001–2003 school years). **Note:** Some calculators are no longer commercially manufactured.

- spreadsheet program
  Any of the following spreadsheet programs may be used: *ClarisWorks™ 4.0* (or higher) for Macintosh or for Windows, *Microsoft® Access 97* and *Microsoft® Excel 97* for Windows (a part of *Microsoft® Office Professional 95* or *Microsoft® Office Professional 97*).

Note: Examples in this course show the TI-83 graphing calculator and the *ClarisWorks™ 4.0* spreadsheet program where applicable.

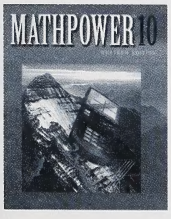
You will need access to a computer (mandatory) and a videocassette player (optional).

Finally, you will need the following resources, which can be purchased locally:

- a mathematical instrument set (compass, protractor, ruler, and triangles)
- a binder in which to respond to the questions asked
- a binder for journal writing

Other Visual Cues

In addition to the Mathematical Process icon described earlier, you will find other visual cues throughout this course. Read the explanations given to discover what each icon prompts you to do.

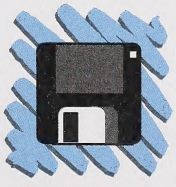


Refer to the textbook.

View a videocassette.

Use the Internet to explore a topic.

Answer the questions in the Assignment Booklet.



Use the Computer Data Bank.

Remember: *Any Internet website address given in this module is subject to change.*

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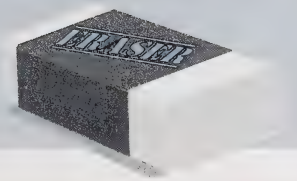
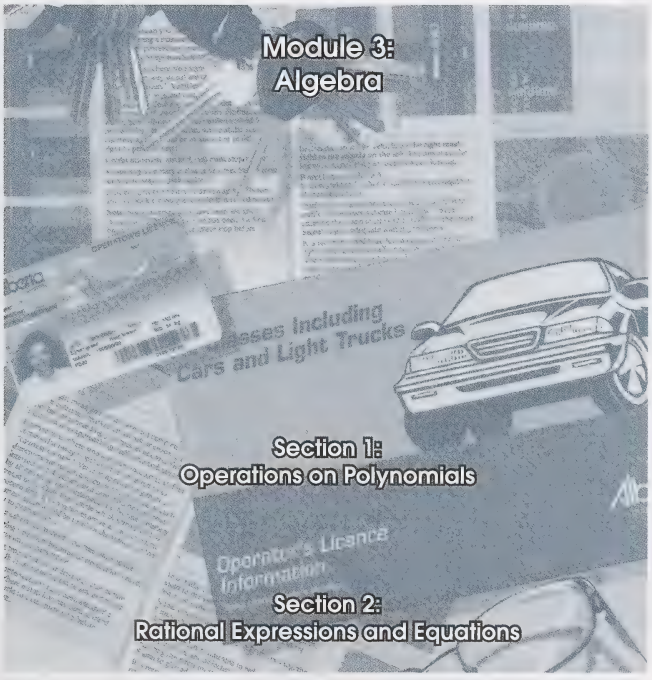


Module Overview

Are you licensed to drive? Before getting your driver's licence, did you take driver training? Driver training teaches you the rules of the road, defensive-driving skills, and basic car maintenance and allows you to practise operating a motor vehicle under the supervision of an experienced instructor. Before getting behind the wheel of a vehicle in driver training, you receive classroom instruction, organized to build each driving skill on the knowledge of previous skills, and you view videos or practise driving in simulators.

Using algebra is similar to learning to operate a motor vehicle. You work with algebra through instruction and practice; you build your algebra skills on the knowledge of previous skills; and you use visualization and diagrams to help solve problems.

In Section 1, you will explore polynomials, use a scientific calculator and paper and pencil to solve problems involving polynomials, and use diagrams or algebra tiles to help visualize polynomials. In Section 2, you will solve rational equations and state any restrictions on the variables. You will also solve problems involving rational expressions and equations.



Evaluation

The document you are presently reading is called a Student Module Booklet. It will show you, step by step, what to do and how to do it.

This module, Algebra, has two sections. Within each section, your work is grouped into activities. Within the activities, there are readings and questions for you to answer. By completing these questions you will construct your own learning, discover mathematical connections, and practise or apply what you have learned. The suggested answers in the Appendix of this Student Module Booklet will provide you with immediate feedback on your progress.

In this module, you will be directed to the accompanying Assignment Booklets. You are expected to complete two Assignment Booklets for this module. Your grading in this module is based in part upon the assignments you submit for evaluation. The mark distribution is as follows:

Assignment Booklet 3A	
Section 1 Assignment	**50 marks**
Assignment Booklet 3B	
Section 2 Assignment	**40 marks**
Final Module Assignment	**10 marks**
TOTAL	**100 marks**

Strategies for Completing a Module

Before you begin, organize your materials: Student Module Booklet, textbook, Assignment Booklet, binder, lined paper, graph paper, graphing calculator, pens, pencils, and so on. Make sure you have a quiet area in which to work, away from distractions. Set up your time schedule.

To achieve success in this module, be sure to read all your instructions carefully and work slowly and systematically through the material. Remember, it's the work you do in this Student Module Booklet that will prepare you for your assignments and final test. Try to set realistic goals for yourself each day; and once you've set them, stick to them. Submit your assignments regularly, and don't forget to review your work before handing it in. Careful work habits will greatly increase your chances for success in Pure Mathematics 10.

Good luck!

Section 1

Operations on Polynomials

Jared is reading a book printed in Braille. Braille is a system of printing used by people who are blind. Raised dots are embossed in paper so that a person may read by touch.

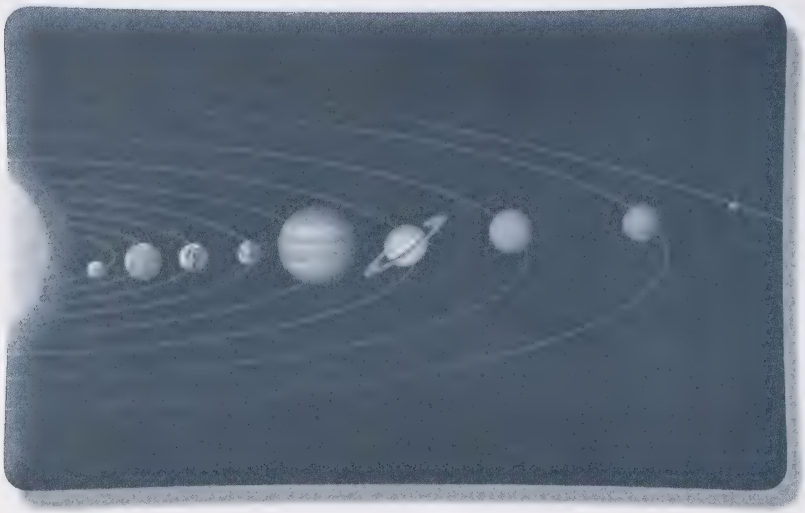
Did you know that the inventor of this system was blind? Louis Braille lost his sight at the age of three as a result of an accident. He became an outstanding teacher of the blind in France. He published his system in 1829 at the age of twenty.

Algebra and Braille are similar in some respects. In Braille, each letter, number, and punctuation mark is represented by one to six dots in a cell (or letter space) that is two dots wide and three dots high. In algebra, unknown numbers are represented by letters. It also requires instruction and practice to learn both Braille and algebra.

In this section, you will extend your knowledge of algebra. You will multiply, factor, and divide polynomials.

Activity 1: Multiplying Polynomials

Have you ever investigated the planets in the solar system? A model like the one in the photograph is helpful in comparing the sizes of the planets and their relative distances from the Sun.

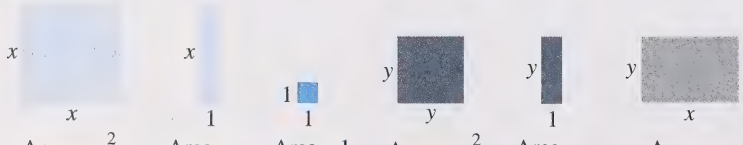


NASA/JPL/CALTECH

In this activity, you will use algebra tiles to model the multiplication of polynomials. Algebra tiles are useful in visualizing the relationship between factors and products.

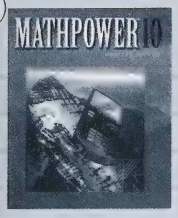
In the *MATHPOWER™ 10* textbook, white tiles are used to represent negative variables and numbers and coloured tiles are used to represent positive variables and numbers. A different colour is used to represent each type of tile used.

In this Student Module Booklet, white tiles are used to represent negative variables and numbers and coloured tiles are used to represent positive variables and numbers. The tiles used in this Student Module Booklet differ in colour from those used in the textbook. All of the positive tiles you will be using throughout this module are as follows:



Mathematical Process

- Communication
- [x] Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- [x] Visualization



Turn to "INVESTIGATING MATH" on page 98 of *MATHPOWER™ 10* and read "Algebra Tiles."

1. Answer the following questions on pages 98 and 99 of the textbook:

   a. question 1 of Investigation 1, "Representing Expressions With Tiles"
   b. question 1 of Investigation 2, "The Zero Principle"
   c. questions 1 and 2 of Investigation 4, "Designing Algebra Tiles"

Compare your responses with those in the Appendix, Section 1: Activity 1.

Mathematical Process

- Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- ☒ Visualization

monomial: a number, a variable, or a product of a number and one or more variables

polynomial: an algebraic expression formed by adding or subtracting monomials

Multiplying a Monomial and a Polynomial

You can use algebra tiles and diagrams to illustrate multiplying a **monomial** and a **polynomial**.

Example

Model the expression and state the product for each of the following:

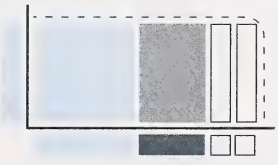
a. $2(x+y+1)$ **b.** $x(x+y-2)$

Solution

Step 1: Model the expression. Use the sides of the algebra tiles to mark off the dimensions along two perpendicular segments; then use the marks as guides and make a rectangle with algebra tiles. **Note:** Start at the intersection of the two perpendicular segments, and use as few tiles as possible.

a.

b.

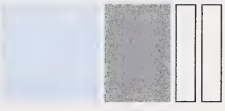


Step 2: Find the product by counting the algebra tiles that make up the rectangle.

a.

$\therefore\ 2(x+y+1)=2x+2y+2$

b.



$\therefore\ x(x+y-2)=x^2+xy-2x$

2. Use algebra tiles to find the product of each of the following:

a. $3(x+5)$ **b.** $2x(x+y-3)$

Compare your responses with those in the Appendix, Section 1: Activity 1.

Now that you have used algebra tiles to visualize the process of multiplying a monomial and a polynomial, you are ready to multiply a monomial and a polynomial using paper and pencil or mental math.

Multiplying a trinomial by a monomial in algebra is similar to multiplying a three-digit number by a one-digit number in arithmetic. Work through the following example to see how these multiplications are similar.

Example

Find the product of each of the following:

a.
$$\begin{array}{r} 312 \\ \times\ \ 4 \\ \hline \end{array}$$

b.
$$\begin{array}{r} x^2+3x-2 \\ \times\qquad 4x \\ \hline \end{array}$$

Solution

a.
$$\begin{array}{r} 312 \\ \times\ \ 4 \\ \hline 1248 \end{array}$$

b.
$$\begin{array}{r} x^2+3x-2 \\ \times\qquad 4x \\ \hline 4x^3+12x^2-8x \end{array}$$

Recall that in order to find the product of the arithmetic problem, you must multiply each digit of 312 by the 4. Notice that in the algebraic problem, as in the arithmetic problem, each term of x^2+3x-2 is multiplied by $4x$.

Mathematical Process

- Communication
- [x] Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization

expand: change the factor form of an expression to a polynomial

Example

Expand each of the following expressions:

a. $2(x+y+1)$

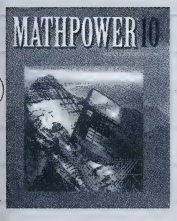
b. $-x(x+y-2)$

Solution

a.
$$\begin{aligned} 2(x+y+1) &= 2(x+y+1) \\ &= 2(x)+2(y)+2(1) \\ &= 2x+2y+2 \end{aligned}$$

b.
$$\begin{aligned} -x(x+y-2) &= -x(x+y-2) \\ &= -x^2-xy+2x \\ &= 2x-xy-x^2 \end{aligned}$$

The last step is optional. However, by writing the polynomial in ascending order instead of descending order, the first term has a positive coefficient.



3. Turn to page 107 of *MATHPOWER™ 10* and answer the following:

a. questions 1, 3, 5, and 9 of "Practice"
b. questions 17, 19, 21, 23, and 25 of "Practice"

Compare your responses with those in the Appendix, Section 1: Activity 1.

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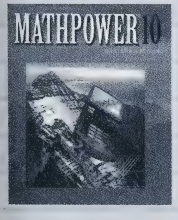
simplify: to perform the required operations and combine like terms



You are now ready to **simplify** expressions using the rules for order of operations. You will recall that multiplication and division are performed before addition and subtraction.

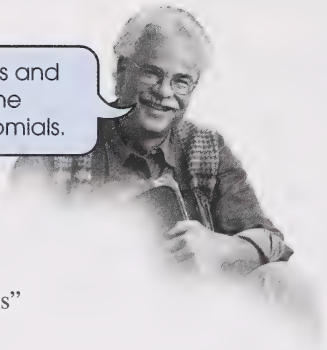
Turn to page 107 of *MATHPOWER™ 10* and work through Example 3. Remember that in order to subtract a polynomial, you must add its opposite. To determine the opposite, multiply each term by -1.

4. Answer the following questions on page 107 of the textbook:

a. questions 11 to 16 of "Practice"
b. questions 26, 28, and 30 of "Practice"

Compare your responses with those in the Appendix, Section 1: Activity 1.

Multiplying Two Binomials



5. Turn to pages 106 and 108 of *MATHPOWER™ 10* and answer the following:

a. questions a. and b. of "Explore: Use Algebra Tiles" on page 106

b. questions 31 and 44 of "Practice" on page 108

c. questions 1 and 5 of "Inquire" on page 106

Compare your responses with those in the Appendix, Section 1: Activity 1.

Now that you have used algebra tiles and diagrams to visualize the process of multiplying two binomials, you are ready to multiply two binomials using paper and pencil or mental math.

Multiplying two binomials in algebra is similar to multiplying two two-digit numbers in arithmetic. Work through the following example to see how these multiplications are similar.

Example

Find the product of each of the following:

a.
$$\begin{array}{r} 12 \\ \times\ 14 \\ \hline \end{array}$$

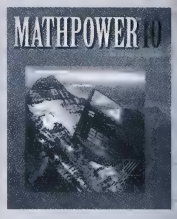
b.
$$\begin{array}{r} x+3 \\ \times\ x-5 \\ \hline \end{array}$$

Solution

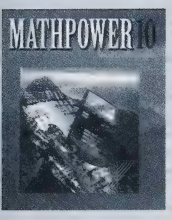
a.
$$\begin{array}{r} 12 \\ \times\ 14 \\ \hline 48 \\ 12 \\ \hline 168 \end{array}$$

b.
$$\begin{array}{r} x+3 \\ \times\ x-5 \\ \hline -5x-15 \\ x^2+3x \\ \hline x^2-2x-15 \end{array}$$

Mathematical Process
- Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- [x] Visualization

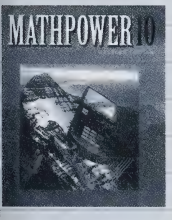


Mathematical Process
- Communication
- [x] Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization



Mathematical Process

- Communication
- Connection
- Estimation
- [x] Mental Math
- Problem Solving
- Reasoning
- Technology
- [x] Visualization

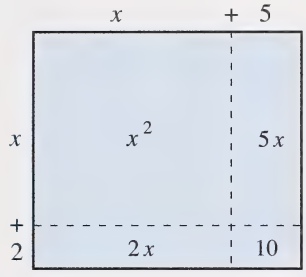


Notice that in the arithmetic problem, 12 is multiplied by each digit of 14, keeping in mind place value. Likewise, in the algebraic problem, each term in $x+3$ is multiplied by each term in $x-5$, keeping in mind like terms.

You can also use the horizontal method when multiplying two binomials. Turn to page 107 of *MATHPOWER™ 10* and work through Example 1.

Notice in Solution 2 that the FOIL acronym (**F**irst, **O**utside, **I**nside, **L**ast) helps you to quickly compute the product of two binomials.

6. Explain how finding the area of the rectangle in the following diagram is related to using the FOIL acronym to find the product of $(x+5)$ and $(x+2)$.



$$(x+5)(x+2)=\underbrace{x(x)}_{\text{F}}+\underbrace{x(2)}_{\text{O}}+\underbrace{5(x)}_{\text{I}}+\underbrace{5(2)}_{\text{L}}$$
$$=x^2+2x+5x+10$$
$$=x^2+7x+10$$

7. Turn to page 108 of *MATHPOWER™ 10* and answer the following:

a. questions 33, 35, 37, 39, and 41 of "Practice"
b. questions 45, 47, 49, 51, 53, and 55 of "Practice"

Compare your responses with those in the Appendix, Section 1: Activity 1.

Example

Simplify $a+5-3(a+2)(a-6)$.

Solution

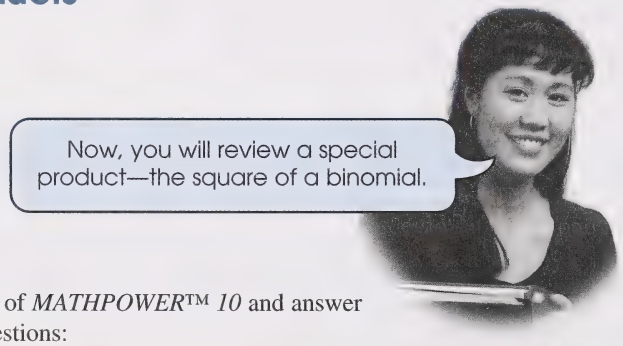
$$\begin{aligned} a+5-3(a+2)(a-6) &= a+5-3\left(a^2-6a+2a-12\right) \\ &= a+5-3\left(a^2-4a-12\right) \\ &= a+5-3a^2+12a+36 \\ &= -3a^2+a+12a+5+36 \\ &= -3a^2+13a+41 \\ &= 41+13a-3a^2 \end{aligned}$$

Note: It is helpful to expand $(a+2)(a-6)$ first.

8. Turn to page 108 of *MATHPOWER™ 10* and answer questions 63, 65, and 67 of "Practice."

Compare your responses with those in the Appendix, Section 1: Activity 1.

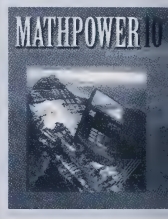
Special Products



9. Turn to page 110 of *MATHPOWER™ 10* and answer the following questions:

a. "Explore: Look for Patterns" at the top of page 110
b. questions 1 to 4 of "Inquire"

Compare your responses with those in the Appendix, Section 1: Activity 1.



Mathematical Process

- Communication
- Connection
- Estimation
- [x] Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization

You can use the pattern you have just discovered to mentally compute the square of a binomial.

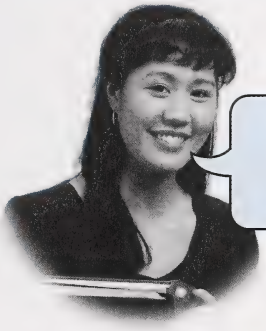
Turn to page 111 of *MATHPOWER™ 10* and work through Example 1.

10. Answer the following questions on pages 112 and 113 of the textbook:

a. questions 1 to 4 of "Practice"
b. questions 10, 12, 14, and 16 of "Practice"
c. questions 18, 20, 22, and 24 of "Practice"
d. questions 54.a., 54.c., 54.e., and 54.g. of "Applications and Problem Solving"

Compare your responses with those in the Appendix, Section 1: Activity 1.

conjugate binomials: two binomials of the form (a + b) and (a – b)

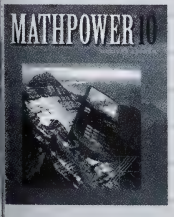


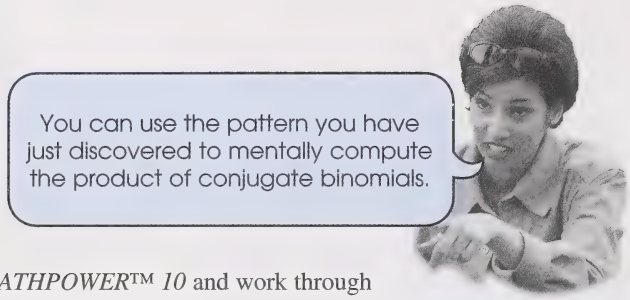
Now, you will review another special product: the product of **conjugate binomials**.

11. Turn to pages 110 and 111 of *MATHPOWER™ 10* and answer the following questions:

a. "Explore: Look for Patterns" at the bottom of page 110
b. questions 1 to 5 of "Inquire" on page 111

Compare your responses with those in the Appendix, Section 1: Activity 1.



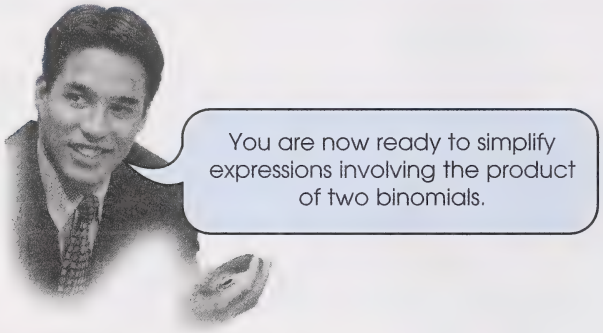


Turn to page 111 of *MATHPOWER™ 10* and work through Example 2.

12. Answer the following questions on pages 112 and 113 of the textbook:

- **a.** questions 5 to 8 of "Practice"
- **b.** questions 11, 13, and 15 of "Practice"
- **c.** questions 17, 19, 21, and 23 of "Practice"
- **d.** questions 55.a., 55.c., 55.e., and 55.f. of "Applications and Problem Solving"

Compare your responses with those in the Appendix, Section 1: Activity 1.



Turn to page 111 of *MATHPOWER™ 10* and work through Example 3.

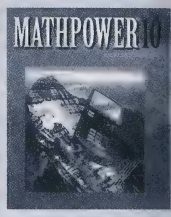
13. Answer the following questions on page 112 of the textbook:

- **a.** questions 28 to 30 of "Practice"
- **b.** questions 35, 37, and 39 of "Practice"

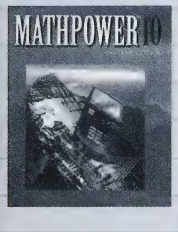
Compare your responses with those in the Appendix, Section 1: Activity 1.

Mathematical Process

- Communication
- Connection
- Estimation
- [x] Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization



Mathematical Process
- Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- [x] Visualization



Multiplying a Binomial and a Trinomial

You can use algebra tiles or diagrams to model the multiplication of a binomial and a trinomial.

14. Turn to page 108 of *MATHPOWER™ 10* and answer question 68 of "Practice."

Compare your responses with those in the Appendix, Section 1: Activity 1.

Now that you have visualized the process of multiplying a binomial and a trinomial, you are ready to multiply a binomial and a trinomial using paper and pencil or mental math.

Multiplying a binomial and a trinomial in algebra is similar to multiplying a two-digit number and a three-digit number in arithmetic. Work through the following example to see how these multiplications are similar.

Mathematical Process
- Communication
- [x] Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization

Example

Find the product of each of the following:

a. $\begin{array}{r} 212 \\ \times\ 32 \\ \hline \end{array}$

b. $\begin{array}{r} x^2 - x + 3 \\ \times\quad x - 5 \\ \hline \end{array}$

Solution

a.
$$\begin{array}{r} 212 \\ \times\ 32 \\ \hline 424 \\ 636 \\ \hline 6784 \end{array}$$

b.
$$\begin{array}{r} x^2 - x + 3 \\ \times\quad x - 5 \\ \hline -5x^2 + 5x - 15 \\ x^3 - x^2 + 3x \\ \hline x^3 - 6x^2 + 8x - 15 \end{array}$$

Notice that in the arithmetic problem, 212 is multiplied by each digit of 32, keeping in mind place value. Likewise, in the algebraic problem, each term in $x^2 - x + 3$ is multiplied by each term in $x - 5$, keeping in mind like terms.

Therefore, when you multiply a trinomial by a binomial, multiply each term in the trinomial by each term in the binomial.

You can also use the horizontal method when multiplying a binomial and a trinomial. Turn to page 107 of *MATHPOWER™ 10* and work through Example 2.

15. Answer questions 69 to 72 of "Practice" on page 108 of the textbook.

Compare your responses with those in the Appendix, Section 1: Activity 1.

Multiplying Three or More Binomials

To find the product of three binomials, multiply two of the binomials and then multiply the product by the third binomial. The process is continued if there are more than three binomials.

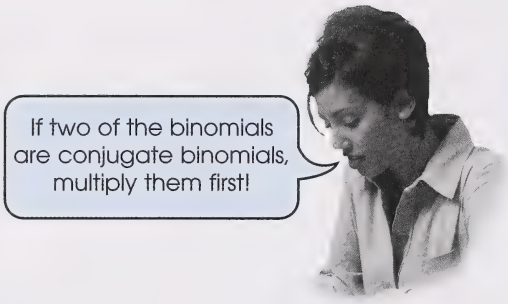
Work through the following example.

Example

Expand and simplify each of the following:

a. $(x-3)(x+3)^2$

b. $(y+1)(y-1)(y+3)^2$



Solution

a. $(x-3)(x+3)^2 = (x-3)(x+3)(x+3)$

$= \left(x^2-9\right)(x+3)$

$= x^2(x)+x^2(3)+(-9)(x)+(-9)(3)$

$= x^3+3x^2-9x-27$

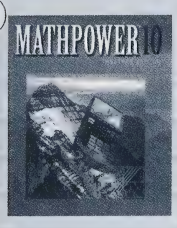
b. $(y+1)(y-1)(y+3)^2 = (y+1)(y-1)(y+3)(y+3)$

$= \left(y^2-1\right)\left(y^2+6y+9\right)$

$= y^2\left(y^2+6y+9\right)+(-1)\left(y^2+6y+9\right)$

$= y^4+6y^3+9y^2-y^2-6y-9$

$= y^4+6y^3+8y^2-6y-9$



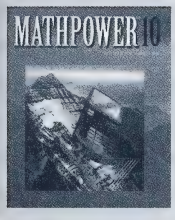
16. Turn to pages 112 and 113 of *MATHPOWER™ 10* and answer the following:

a. question 42 of "Practice"
b. questions 51.a. and 56.a. of "Applications and Problem Solving"

Compare your responses with those in the Appendix, Section 1: Activity 1.

Multiplying the Cube of a Binomial

To find the cube of a binomial, find the square of the binomial and then multiply the square by the binomial.



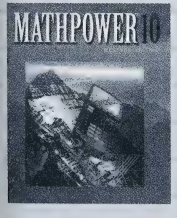
Turn to page 111 of *MATHPOWER™ 10* and work through Example 4.

17. Answer questions 43, 45, and 47 of "Practice" on page 112 of the textbook.

Compare your responses with those in the Appendix, Section 1: Activity 1.

Mathematical Process
- Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- [x] Visualization

Multiplying Two Trinomials



18. Turn to page 113 of *MATHPOWER™ 10* and answer question 58.a. of "Applications and Problem Solving."

Compare your response with the one in the Appendix, Section 1: Activity 1.

Now that you have visualized the process of multiplying two trinomials, you are ready to multiply two trinomials using paper and pencil or mental math.

Multiplying two trinomials in algebra is similar to multiplying two three-digit numbers in arithmetic. Work through the following example to see how these multiplications are similar.

Example

Find the product of each of the following:

a. $\begin{array}{r} 112 \\ \times\ 312 \\ \hline \end{array}$

b. $\begin{array}{r} x^2 - x + 3 \\ \times\ x^2 - 2x + 5 \\ \hline \end{array}$

Solution

a.
$$\begin{array}{r} 112 \\ \times\ 312 \\ \hline 224 \\ 112 \\ 336 \\ \hline 34944 \end{array}$$

b.
$$\begin{array}{r} x^2 - x + 3 \\ \times\ x^2 - 2x + 5 \\ \hline 5x^2 - 5x + 15 \\ -2x^3 + 2x^2 - 6x \\ x^4 - x^3 + 3x^2 \\ \hline x^4 - 3x^3 + 10x^2 - 11x + 15 \end{array}$$

Notice that in the arithmetic problem, 112 is multiplied by each digit of 312, keeping in mind place value. Likewise, in the algebraic problem, each term in $x^2 - x + 3$ is multiplied by each term in $x^2 - 2x + 5$, keeping in mind like terms.

Therefore, when you multiply two trinomials, multiply each term in one trinomial by each term in the other trinomial.

You can also use the horizontal method when multiplying two trinomials. Work through the following example.

Example

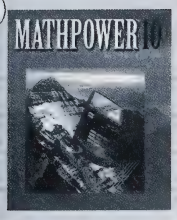
Expand $(x^2 - x + 3)(x^2 - 2x + 5)$.

Solution

$$\begin{aligned} (x^2 - x + 3)(x^2 - 2x + 5) &= x^2(x^2 - 2x + 5) + (-x)(x^2 - 2x + 5) + 3(x^2 - 2x + 5) \\ &= x^4 - 2x^3 + 5x^2 - x^3 + 2x^2 - 5x + 3x^2 - 6x + 15 \\ &= x^4 - 2x^3 - x^3 + 5x^2 + 2x^2 + 3x^2 - 5x - 6x + 15 \\ &= x^4 - 3x^3 + 10x^2 - 11x + 15 \end{aligned}$$

Mathematical Process

- Communication
- [X] Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization



Mathematical Process
- Communication
- Connection
- Estimation
- [x] Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization

19. Turn to page 108 of *MATHPOWER™ 10* and answer questions 73 and 75 of "Practice."

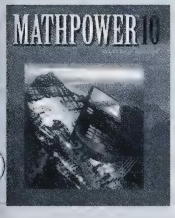
Compare your responses with those in the Appendix, Section 1: Activity 1.

Now Try This

Over the years, you have probably developed many strategies for performing mental computations. You will now discover and apply a strategy for multiplying numbers that differ by 2.

20. Turn to page 97 of *MATHPOWER™ 10* and answer questions 1 to 9 of "Mental Math: Multiplying Two Numbers That Differ by 2."

Compare your responses with those in the Appendix, Section 1: Activity 1.



Finding two numbers given their product and sum is an important skill when you are factoring a trinomial. (You will be doing this in the next activity.)

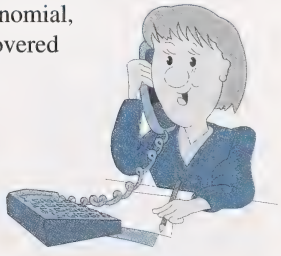
21. Turn to page 97 of *MATHPOWER™ 10* and answer questions 1 to 20 of "Mental Math: Integers."

Compare your responses with those in the Appendix, Section 1: Activity 1.

Mathematical Process
- [x] Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization

Looking Back

In this activity, you reviewed multiplying a polynomial by a monomial, multiplying two binomials, and special products. You then discovered how to multiply a trinomial by a binomial, how to multiply two trinomials, and how to multiply three or more binomials.



Suppose a friend calls you to discuss multiplying polynomials. How would you explain how to multiply two polynomials over the phone? How would you explain how to multiply three polynomials? Record your explanations in your journal, including examples.

Activity 2: Factoring Polynomials

Some metals occur naturally in the earth; other metals are produced by combining two or more basic metals. For example, gold forms naturally and pewter forms by combining tin and copper.

Similarly, some natural numbers are **prime numbers** and other natural numbers are products of two or more prime numbers.

In earlier schooling, you expressed a number as the product of its prime factors. To review this procedure, work through the following example.

Mathematical Process

- Communication
- [X] Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization

prime number: a number with only two factors: 1 and itself

Example

Express 36 as a product of prime factors.

Solution

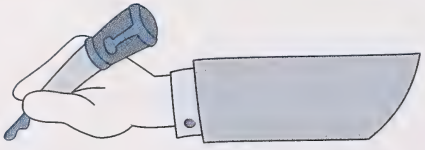
$$36 = 6 \times 6$$
$$= 2 \times 3 \times 2 \times 3$$
$$= 2 \times 2 \times 3 \times 3$$

Note: Usually, the product of prime factors is written in ***ascending order.***

1. Express each of the following numbers as a product of prime factors:

a. 42 **b.** 45 **c.** 100 **d.** 110

Compare your responses with those in the Appendix, Section 1: Activity 2.



common factor: a number or expression that is common to two or more numbers or polynomials

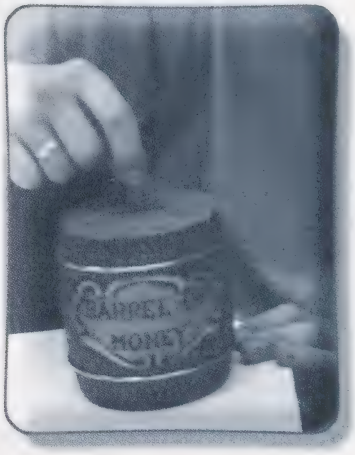
greatest common factor (GCF): the greatest number or expression that is common to two or more numbers or polynomials

Common Factors

In previous schooling, you found the **common factors** and the **greatest common factor** (GCF) of two natural numbers. Work through the following example to refresh your memory.

Example

Lucille has 36 nickels and 48 quarters. She decides to arrange the coins in stacks so that each stack contains the same number of coins and that each stack contains only nickels or only quarters. What is the greatest number of coins each stack can have?



Solution

Step 1: Write the prime factorization of each number.

$$36 = 2 \times 2 \times 3 \times 3$$
$$48 = 2 \times 2 \times 2 \times 2 \times 3$$

Step 2: Arrange the prime factorizations so that the common prime factors of each number are aligned in columns.

$$36 = 2 \times 2 \quad \times 3 \times 3$$
$$48 = 2 \times 2 \times 2 \times 2 \times 3$$

Step 3: To find the greatest common factor (GCF), take one factor from each complete column and multiply.

$$\text{GCF} = 2 \times 2 \times 3$$
$$= 12$$

The greatest number of coins each stack can have is 12.

2. Helmut paid \$78 for several doorknob sets. Eddie bought several doorknob sets for \$117. If every doorknob set was the same price, what is the most that each could have cost?

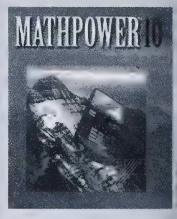
Compare your response with the one in the Appendix, Section 1: Activity 2.

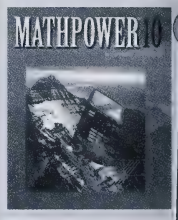
Factoring a Polynomial

You can use algebra tiles to illustrate the process of finding the greatest common factor of the terms of a polynomial.

Mathematical Process

- Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization



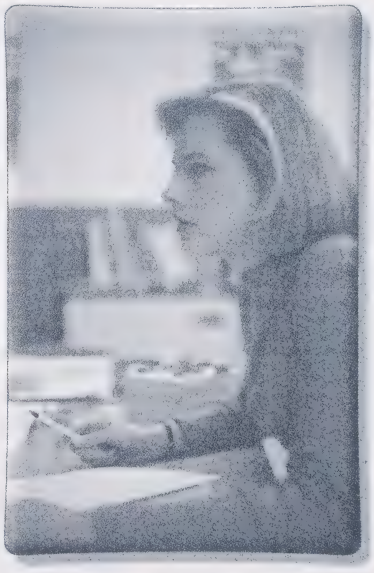


3. Turn to page 118 of *MATHPOWER™ 10* and read "Explore: Use Algebra Tiles"; then copy and complete the table given.

4. Turn to page 119 of *MATHPOWER™ 10* and answer questions 1 and 2 of "Inquire."

Compare your responses with those in the Appendix, Section 1: Activity 2.

To discover what the process "factoring over the integers" means, turn to page 119 of *MATHPOWER™ 10* and read from the red line near the top of the page to Example 1.



5. What does the process "factoring over the integers" mean?

Compare your response with the one in the Appendix, Section 1: Activity 2.

Factoring by Removing a Common Factor

To discover how to find a monomial common factor of a polynomial, turn to page 119 of *MATHPOWER™ 10* and work through Example 1.

6. Answer the following questions on page 120 of the textbook:

a. questions 1 to 6 of "Practice"
b. questions 11, 13, 15, 17, 19, and 21 of "Practice"

Compare your responses with those in the Appendix, Section 1: Activity 2.

To discover how to find a binomial common factor of a polynomial, turn to page 119 of *MATHPOWER™ 10* and work through Example 2.

7. Answer questions 23, 25, 27, and 29 of "Practice" on page 120 of the textbook.

Compare your responses with those in the Appendix, Section 1: Activity 2.

Factoring by Grouping

To discover how to factor by grouping, turn to page 119 of *MATHPOWER™ 10* and work through Example 3.

8. Answer questions 31, 33, and 35 of "Practice" on page 120 of the textbook.

Compare your responses with those in the Appendix, Section 1: Activity 2.

Factoring Trinomials of the Form $x^2 + bx + c$

Turn to page 125 of *MATHPOWER™ 10* and read "Factoring $x^2 + bx + c$."

9. Answer the following questions on pages 125 and 127 of the textbook:

- **a.** questions a. and b. of "Explore: Use Algebra Tiles" on page 125
- **b.** questions 1 to 4 of "Inquire" on page 125
- **c.** question 62 of "Applications and Problem Solving" on page 127

Compare your responses with those in the Appendix, Section 1: Activity 2.

Now that you have visualized how to factor trinomials of the form $x^2 + bx + c$, you are ready to factor these trinomials using paper and pencil or mental math.

Turn to pages 125 and 126 of *MATHPOWER™ 10* and read from the red line to the end of Example 1, working through Example 1.

Notice that in order to factor a trinomial of the form $x^2 + bx + c$, you must find two numbers whose sum is b and whose product is c.

Remember that some trinomials cannot be **factored over the integers**. For example, $x^2 + 3x + 5$ cannot be factored over the integers because no two integers have a sum of 3 and a product of 5.

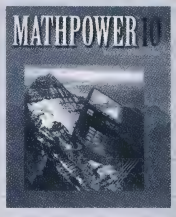
10. Turn to pages 126 and 127 of *MATHPOWER™ 10* and answer the following:

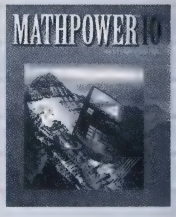
- **a.** questions 9, 11, 13, 15, and 17 of "Practice"
- **b.** questions 19, 21, 23, and 25 of "Practice"
- **c.** questions 27, 29, 31, 33, 35, 37, 39, 41, 49, and 50 of "Practice"
- **d.** questions 65 and 66 of "Applications and Problem Solving"

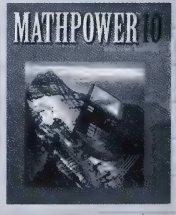
Compare your responses with those in the Appendix, Section 1: Activity 2.

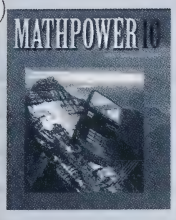
Mathematical Process

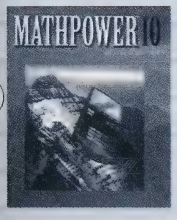
- Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- [x] Visualization

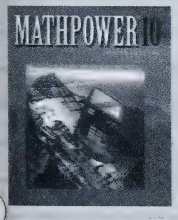








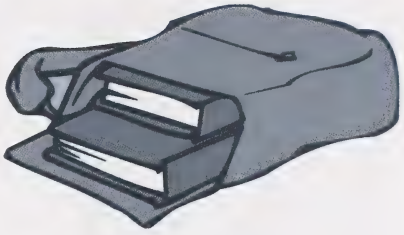




Mathematical Process

- Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- [x] Visualization

Example 1 on page 126 of *MATHPOWER™ 10* shows how to factor trinomials of the form $x^2 + bx + c$, where x is a variable and b and c are integers. You can use this same method to factor trinomials of the form $x^4 + bx^2 + c$, where x is a variable and b and c are integers.



Example

Factor $x^4 - x^2 - 2$.

Solution

Two numbers whose product is -2 and whose sum is -1 are -2 and 1.

$$x^4 - x^2 - 2 = \left(x^2 - 2\right)\left(x^2 + 1\right)$$

11. Turn to page 127 of *MATHPOWER™ 10* and answer questions 67.a. to 67.c. of "Applications and Problem Solving."

Compare your responses with those in the Appendix, Section 1: Activity 2.

You can use algebra tiles and diagrams to help you visualize how to factor a trinomial in two variables.

12. Turn to page 127 of *MATHPOWER™ 10* and answer question 63 of "Applications and Problem Solving."

Compare your responses with those in the Appendix, Section 1: Activity 2.

Now that you have visualized factoring trinomials with two variables, you are ready to use paper and pencil or mental math.

Turn to page 126 of *MATHPOWER™ 10* and work through Example 3.

13. Answer questions 43 to 47 of "Practice" on page 127 of the textbook.

Compare your responses with those in the Appendix, Section 1: Activity 2.

Factoring Trinomials of the form $ax^2 + bx + c$, $a \neq 1$

You will now factor trinomials of the form $ax^2 + bx + c$, where x is a variable; a, b, and c are integers; and $a \neq 1$.

Turn to page 128 of *MATHPOWER™ 10* and read "Factoring $ax^2 + bx + c$, $a \neq 1$."

14. Answer the following questions on page 128 of the textbook:

a. questions a. and b. of "Explore: Use Algebra Tiles"
b. questions 1 to 3 of "Inquire"

Compare your responses with those in the Appendix, Section 1: Activity 2.

When you want to factor a trinomial of the form $ax^2 + bx + c$, where $a \neq 1$, you should first attempt to remove a common factor. Sometimes, in trinomials of the form $ax^2 + bx + c$, the coefficient of the x^2-term is a common factor.

Turn to page 126 of *MATHPOWER™ 10* and work through Example 2.

15. Answer questions 51, 53, 55, 57, and 59 of "Practice" on page 127 of the textbook.

Compare your responses with those in the Appendix, Section 1: Activity 2.

If there is no common factor to remove, you can use one of two methods to factor the trinomial:

- using the guess and check strategy
- breaking up the middle term of the trinomial and then factoring by grouping

If there is a common factor, remove it; then factor the trinomial factor.

Remember: *Some trinomials cannot be factored over the integers!*

Turn to page 128 of *MATHPOWER™ 10* and read from the red line to the end of Example 3 on page 130. Be sure to work through all the examples carefully.

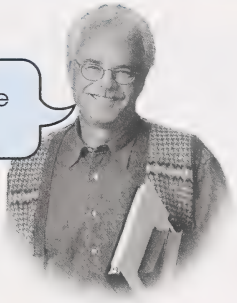
Note: In Example 3, two steps were used to factor the trinomial. First, the common factor was removed; then the middle term of the trinomial factor was broken into two parts and factored by grouping. Alternatively, the guess and check strategy could have been used in the second step.

16. Answer the following questions on pages 130 and 131 of *MATHPOWER™ 10*.

a. questions 7, 9, and 11 of "Practice"
b. questions 23, 25, 27, 29, 31, 33, 35, 37, and 39 of "Practice"
c. questions 54 and 55 of "Applications and Problem Solving"

Compare your responses with those in the Appendix, Section 1: Activity 2.

Mathematical Process
- Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- [x] Visualization



17. Turn to page 131 of *MATHPOWER™ 10* and answer question 52 of "Applications and Problem Solving."

Turn to page 130 of *MATHPOWER™ 10* and work through Example 4.

18. Answer questions 41, 43, 45, 47, and 49 of "Practice" on page 131 of the textbook.

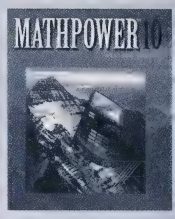
Compare your responses with those in the Appendix, Section 1: Activity 2.

Factoring the Difference of Squares

A binomial written in the form $a^2 - b^2$ is called a **difference of squares**. It is the result of multiplying the conjugate binomials $(a+b)$ and $(a-b)$.

difference of squares: a binomial of the form $a^2 - b^2$

Turn to page 133 of *MATHPOWER™ 10* and work through Examples 1 and 2.



19. Answer questions 27, 30, 31, 38, 40, 41, 43, and 44 of "Practice" on page 133 of the textbook.

Compare your responses with those in the Appendix, Section 1: Activity 2.

Factoring Perfect Square Trinomials

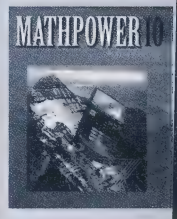
The trinomial that results from squaring a binomial is called a **perfect square trinomial**.

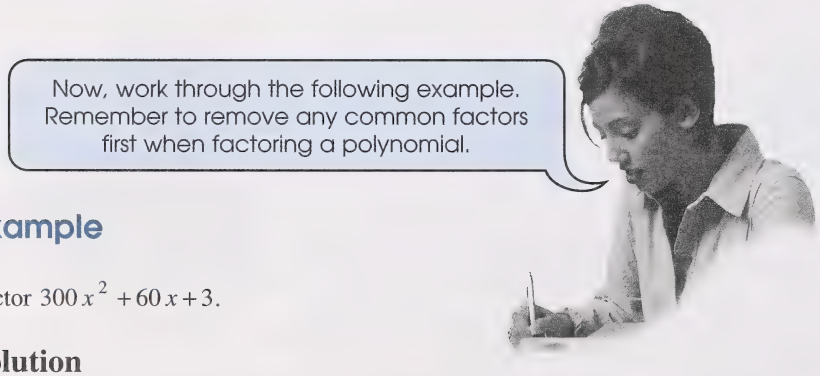
perfect square trinomial: a trinomial of the form $a^2 + 2ab + b^2$

A trinomial is a perfect square trinomial if the following conditions are met:

- The first and last terms are perfect squares.
- The middle term is twice the product of the square roots of the first and last terms.

Turn to page 133 of *MATHPOWER™ 10* and work through Example 3.



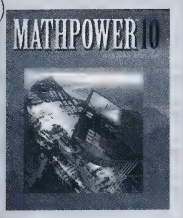


Example

Factor $300x^2 + 60x + 3$.

Solution

$$\begin{aligned} 300x^2 + 60x + 3 &= 3\left(100x^2 + 20x + 1\right) \\ &= 3\left[(10x)^2 + 2(10x)(1) + 1^2\right] \\ &= 3(10x+1)^2 \end{aligned}$$



Mathematical Process
- Communication
- [X] Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization

20. Turn to pages 133 and 134 of *MATHPOWER™ 10* and answer the following:

a. question 45 of "Applications and Problem Solving"
b. questions 28, 29, 32, 34, 37, and 39 of "Practice"
c. question 55 of "Applications and Problem Solving"

Compare your responses with those in the Appendix, Section 1: Activity 2.

Now Try This

CORBIS/STUART WESTMORLAND

Have you ever visited Butchart Gardens on Vancouver Island? Butchart Gardens is Canada's largest privately owned garden.

If you have access to the Internet, visit the following website for historical information and scenes of Butchart Gardens:

http://butchartgardens.bc.ca/butchart/

In Canada, Butchart Gardens was made from an abandoned quarry. Similarly, in England, Gwennap Pit was made from a collapsed tin mine.

Turn to "CONNECTING MATH AND ARCHITECTURE" on page 138 of *MATHPOWER™ 10* and read "Gwennap Pit."

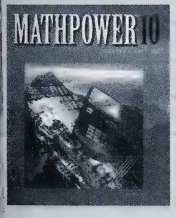
21. Answer questions 1 to 5 of Investigation 1, "Writing Expressions for Areas" on pages 138 and 139 of the textbook.

Compare your responses with those in the Appendix, Section 1: Activity 2.

Looking Back

In this activity, you factored natural numbers and polynomials.

Read the cartoon on page 142 of *MATHPOWER™ 10*. Suppose you had to explain the meaning of *factor* to Peppermint Patty. Record your explanation in your journal.



Mathematical Process
- [X] Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization

Activity 3: Dividing Polynomials

There are many opposite actions in life. For example, opening an umbrella is the opposite of closing an umbrella.

Likewise, there are many opposite operations in mathematics. For example, multiplying and dividing are opposite operations.

Several mathematical terms are used when discussing multiplication and division.

In the multiplication problem $3 \times 4 = 12$, the **product** 12 is said to be a **multiple** of 3 and 3 is said to be a **factor** of 12.

In the division problem $12 \div 3 = 4$, the **dividend** 12 is said to be **divisible** by 3 and 3 is said to be a **divisor** of 12. Because 3 divides evenly into 12, the **quotient** 4 is a whole number. In other words, the **remainder** is zero.

Mathematical Process

- Communication
- [x] Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization

You can use division to check if a number is a factor of another number. When a number divides evenly into another number, it is a factor of the number.

Example

Answer the following:

a. Is 4 a factor of 252?

b. Is 5 a factor of 771?

Solution

Use division to determine whether the first number is a factor of the second number.

a.
$$\begin{array}{r} 63 \\ 4\overline{)252} \\ \underline{24} \\ 12 \\ \underline{12} \\ 0 \end{array}$$

Because 4 divides evenly into 252, it is a factor of 252.

b.
$$\begin{array}{r} 154 \\ 5\overline{)771} \\ \underline{5} \\ 27 \\ \underline{25} \\ 21 \\ \underline{20} \\ 1 \end{array}$$
or
$$\begin{array}{r} 154.2 \\ 5\overline{)771.0} \\ \underline{5} \\ 27 \\ \underline{25} \\ 21 \\ \underline{20} \\ 10 \\ \underline{10} \\ 0 \end{array}$$

Because 5 does not divide evenly into 771, it is not a factor of 771.

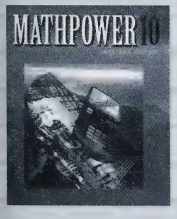
1. Use division to determine whether 3 is a factor of each of the following numbers:

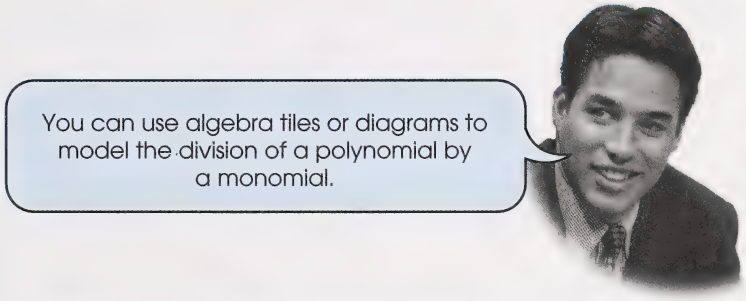
a. 1306 **b.** 9858 **c.** 5841

Compare your responses with those in the Appendix, Section 1: Activity 3.

Dividing a Polynomial by a Monomial

Mathematical Process
- Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- [x] Visualization





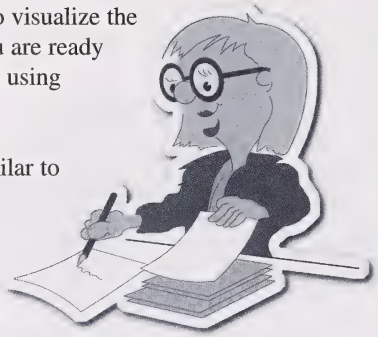
Turn to page 148 of *MATHPOWER™ 10* and read "Dividing Polynomials."

2. Answer the following questions on page 148 of the textbook:

a. questions a. and b. of "Explore: Use Algebra Tiles"
b. questions 1 and 2 of "Inquire"

Compare your responses with those in the Appendix, Section 1: Activity 3.

Now that you have used algebra tiles and diagrams to visualize the process of dividing a polynomial by a monomial, you are ready to review how to divide a polynomial by a monomial using paper and pencil or mental math.



Dividing a trinomial by a monomial in algebra is similar to dividing a three-digit number by a one-digit number in arithmetic. Work through the following example to see how these divisions are similar.

Example

Divide each of the following:

a. $3\overline{)936}$

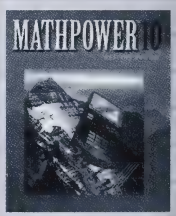
b. $3\overline{)6x^2-3x+9}$

Solution

a.
$$\begin{array}{r} 312 \\ 3\overline{)936} \\ \underline{9} \\ 03 \\ \underline{3} \\ 06 \\ \underline{6} \\ 0 \end{array}$$

b.
$$\begin{array}{r} 2x^2 \quad -x+3 \\ 3\overline{)6x^2-3x+9} \\ \underline{6x^2} \\ 0x^2-3x \\ \underline{0x^2-3x} \\ 0x+9 \\ \underline{0x+9} \\ 0 \end{array}$$

Turn to page 148 of *MATHPOWER™ 10* and read from the red line to the red line on page 149, working through Example 1.

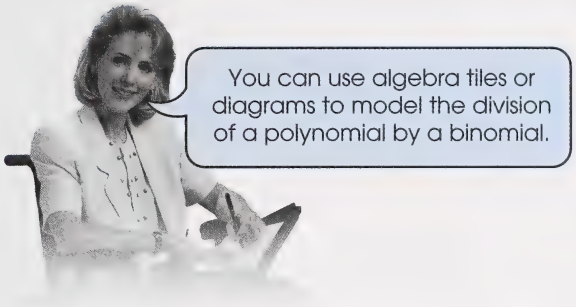


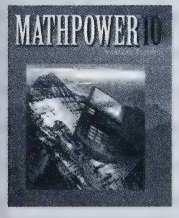
3. Answer questions 11 to 17 of "Practice" on page 152 of the textbook. State any restrictions on the variables.

Compare your responses with those in the Appendix, Section 1: Activity 3.

Dividing a Polynomial by a Binomial

Mathematical Process
- Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- [x] Visualization



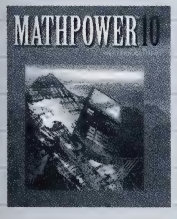


4. Turn to page 153 of *MATHPOWER™ 10* and answer question 67 of "Applications and Problem Solving."

Compare your responses with those in the Appendix, Section 1: Activity 3.

Now that you have used algebra tiles and diagrams to visualize the process of dividing a polynomial by a binomial, you are ready to divide a polynomial by a binomial using paper and pencil or mental math.

Turn to page 149 of *MATHPOWER™ 10* and read from the red line to the red line on page 150, working through each example carefully.

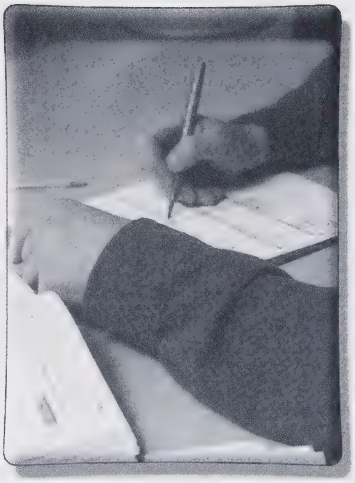


5. Answer questions 19, 21, 23, 25, 27, 29, 31, 33, 35, and 37 of "Practice" on page 152 of the textbook. State any restrictions on the variables.

6. Answer question 68 of "Applications and Problem Solving" on page 153 of the textbook.

Compare your responses with those in the Appendix, Section 1: Activity 3.

In arithmetic, when the divisor is not a factor of the dividend, there will be a remainder. The remainder may be expressed as a fraction of the divisor.



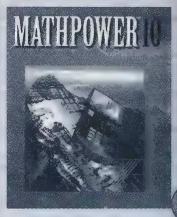
For example,

$$3\overline{)14}\quad 4$$
$$\underline{12}$$
$$2$$

$$\therefore\ 14 \div 3 = 4\ \text{R2 or } 14 \div 3 = 4\frac{2}{3}$$

The same conventions are used for dividing polynomials.

Turn to page 150 of *MATHPOWER™ 10* and read from the red line to the red line on page 151. Work through each example carefully.



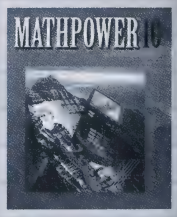
7. Answer questions 47, 49, 51, 53, and 55 of "Practice" on page 152 of the textbook. State any restrictions on the variables.

Compare your responses with those in the Appendix, Section 1: Activity 3.

Using Function Notation

Because the value of a polynomial containing the variable x depends on the value of x, the polynomial can be named using function notation.

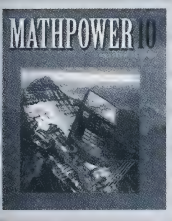
Turn to page 151 of *MATHPOWER™ 10* and read the text between Example 3 and Example 4. Then work through Example 4.



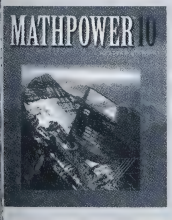
8. Answer questions 57, 59, 61, and 63 of "Practice" on page 152 of the textbook. State any restrictions on the variable in each division problem.

Compare your responses with those in the Appendix, Section 1: Activity 3.

Mathematical Process
- Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- [x] Reasoning
- Technology
- Visualization

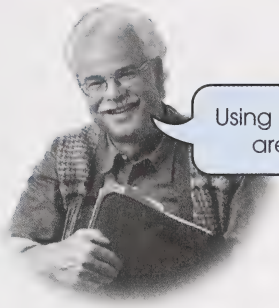


Mathematical Process
- Communication
- Connection
- Estimation
- Mental Math
- [x] Problem Solving
- Reasoning
- Technology
- Visualization



Mathematical Process
- [x] Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization

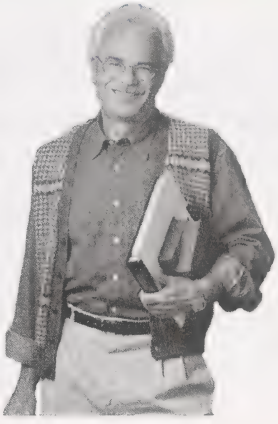
Now Try This



9. Turn to page 109 of *MATHPOWER™ 10* and answer "LOGIC POWER."

Compare your response with the one in the Appendix, Section 1: Activity 3.

Diagrams can be useful for simplifying and solving problems of many kinds, including networking, scheduling, and navigational problems.



Turn to "PROBLEM SOLVING" on page 114 of *MATHPOWER™ 10* and read "Use a Diagram."

10. Answer question 5 of "Applications and Problem Solving" on page 115 of the textbook.

Compare your response with the one in the Appendix, Section 1: Activity 3.

Looking Back

In this activity, you divided polynomials. In your journal, record some of the ways the division of polynomials compares to the division of whole numbers. Also, record some of the ways the division of polynomials differs from the division of whole numbers.

Follow-up Activities

If you had difficulties understanding the concepts and skills in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts and skills, it is recommended that you do the Enrichment. You may decide to do both.

Extra Help

When you perform multiplication and division in arithmetic, it is important that you correctly line up the digits according to place value. Use zeros as placeholders.

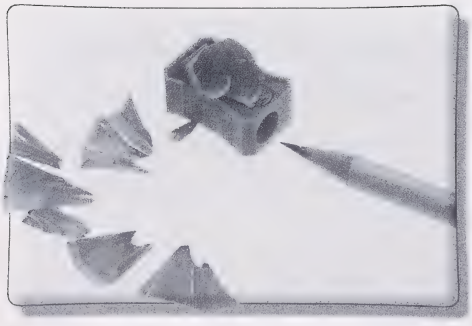
$$\begin{array}{r} 344 \\ \times\ 17 \\ \hline 2308 \\ 344 \\ \hline 5748 \end{array}$$

$$\begin{array}{r} 208 \\ 14\overline{)2912} \\ 28 \\ \hline 11 \\ 0 \\ \hline 112 \\ 112 \\ \hline 0 \end{array}$$

When you multiply and divide polynomials, ensure that the polynomials are written so that the exponents of the variables are in **descending order**. When there are missing terms in the dividend, include them with zero as the coefficient. Also, when you perform multiplication and division, it is important that you correctly line up the like terms.

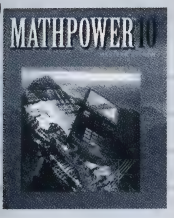
$$\begin{array}{r} x^2+3x-5 \\ \times\qquad x+2 \\ \hline 2x^2+6x-10 \\ x^3+3x^2-5x \\ \hline x^3+5x^2+\ x-10 \end{array}$$

$$\begin{array}{r} 4x^2-3x\ -6 \\ x-2\overline{)4x^3-11x^2+0x+12} \\ 4x^3-\ 8x^2 \\ \hline -3x^2+0x \\ -3x^2+6x \\ \hline -6x+12 \\ -6x+12 \\ \hline 0 \end{array}$$



Mathematical Process

- Communication
- [x] Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization



1. Turn to page 140 of *MATHPOWER™ 10* and answer questions 21 and 22 of "Review."

2. Turn to page 202 of *MATHPOWER™ 10* and answer questions 1 to 4 of "Chapter Check."

Compare your responses with those in the Appendix, Section 1: Follow-up Activities, Extra Help.

Enrichment

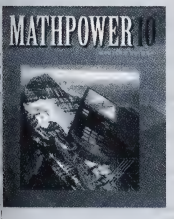
Short division is a shortcut for dividing whole numbers in arithmetic. In short division, most of the steps are done mentally. Compare the steps between long division and short division when 282 is divided by 6.

Long Division

$$\begin{array}{r} 47 \\ 6\overline{)282} \\ \underline{24} \\ 42 \\ \underline{42} \\ 0 \end{array}$$

Short Division

$$\begin{array}{r} 4\ 7 \\ 6\overline{)28^{4}2} \end{array}$$



Synthetic division is a shortcut for dividing polynomials when the divisor is $x-b$. Turn to "INVESTIGATING MATH" on page 154 of *MATHPOWER™ 10* and read Investigation 1, "Dividing Polynomials by $x-b$." Compare the steps between long division and synthetic division when $4x^3-11x^2+12$ is divided by $x-2$.

1. Answer questions 1, 4, 6, 8 and 9 of Investigation 1, "Dividing Polynomials by $x-b$" on page 154 of the textbook.

Compare your responses with those in the Appendix, Section 1: Follow-up Activities, Enrichment.

In question 1, you performed synthetic division using a divisor of the form $x-b$, where b is positive.

What if the sign in the divisor is positive? In the divisor $x+3$, the variable b is negative because $x+3=x-(-3)$. Therefore, $x-b=x-(-3)$ and $b=-3$.

2. Turn to page 154 of *MATHPOWER™ 10* and answer questions 2, 3, 5, and 7 of Investigation 1, "Dividing Polynomials by $x-b$."

Compare your responses with those in the Appendix, Section 1: Follow-up Activities, Enrichment.

So far, every division problem that you have computed using synthetic division has had no remainder.

Work through the following example, and compare the steps between long division and synthetic division. Notice that the remainder is expressed as a fraction of the divisor.

Example

Calculate $\left(x^2-2x-9\right)\div(x-4)$ using the following:

a. long division **b.** synthetic division

Solution

a.
$$\begin{array}{r} x+2 \\ x-4\overline{)\,x^2-2x-9} \\ \underline{x^2-4x} \\ 2x-9 \\ \underline{2x-8} \\ -1 \end{array}$$

$$\therefore \left(x^2-2x-9\right)\div(x-4) = x+2+\frac{-1}{x-4}$$
$$= x+2-\frac{1}{x-4}$$

b.
$$\begin{array}{r|rrr} 4 & 1 & -2 & -9 \\ & & 4 & 8 \\ \hline & 1 & 2 & -1 \end{array}$$

$$\therefore \left(x^2-2x-9\right)\div(x-4) = x+2+\frac{-1}{x-4}$$
$$= x+2-\frac{1}{x-4}$$

3. Turn to page 154 of *MATHPOWER™ 10* and answer questions 11, 13, 15, and 17 of Investigation 1, "Dividing Polynomials by $x-b$."

Compare your responses with those in the Appendix, Section 1: Follow-up Activities, Enrichment.

Conclusion

In this section, you extended your skills with polynomials. You multiplied combinations of monomials, binomials, and trinomials; and you squared binomials and multiplied conjugate binomials. Then, you factored polynomials by removing common factors, by grouping terms, and by writing trinomials and differences of squares as products of binomials. Finally, you divided polynomials by both monomials and binomials.

Throughout this section, you manipulated the algebraic symbols within the polynomials. It is these symbols that visually represent the abstract notions of this section. Have you ever wondered how difficult it would be to master mathematics if you could not see these symbols?

Louis Braille, who was born in 1809 and who lost his sight in an accident at the age of three, published his system for representing letters and numbers in 1829. His system, based on raised dots within "Braille cells," has opened up countless educational opportunities for the blind.

If you have access to the Internet, do a little research on Braille. You may find the following website interesting:

http://www.langara.bc.ca/cba/index.html

After doing a little research on Braille, can you decipher the following?

Assignment

Turn to Assignment Booklet 3A and complete the assignment for Section 1.

Section 2

Rational Expressions and Equations

The Queen Charlotte Islands, consisting of approximately 250 islands and totaling over 9000 km^2 in area, is a unique archipelago lying off the coast of British Columbia. The two largest islands in the group are Graham and Moresby. Graham Island, itself, is over two-thirds of the total area. Because of the movement of Earth's crust, the Queen Charlottes are thought to have migrated from the South Pacific to their present location. Spared from the ravages of the last ice age, these islands are home to many plant and animal species not found anywhere else. The old-growth cedar and spruce forests have been protected by the establishment of a national park, called South Moresby, which comprises about one-seventh of the total area of the Queen Charlotte Islands.

Notice how often numbers and fractions occur in written descriptions and how quickly your mind translates these into mental images. Fractions are a natural part of mathematics and are a subset of a larger class called rational expressions.

In this section, you will define rational expressions; add, subtract, multiply, and divide rational expressions; and use rational expressions in equations to solve real-world problems.

Activity 1: Simplifying Rational Expressions

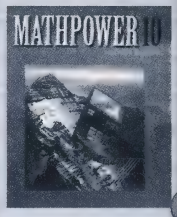
Frank is taking a course on keyboarding. His teacher uses the following formula to calculate typing speed:

$$T = \frac{w - 10e}{t}$$

In this formula, T is the typing speed (in words per minute), w is the number of words typed in t minutes, and e is the number of errors made.

The expression $\frac{w-10e}{t}$ is an example of a **rational expression**.

rational expression: a fraction with a variable in the numerator and/or denominator



To view more examples of rational expressions, turn to page 156 of *MATHPOWER™ 10* and read "Simplifying Rational Expressions."

Remember, division by zero is undefined. If a variable appears in the denominator of a rational expression, you cannot replace the variable by a value that makes the denominator zero.

For example, in the expression $\frac{2x^2+3x+1}{2x+1}$, the denominator $2x+1$ cannot equal zero.

$$\therefore\ 2x+1 \neq 0$$
$$2x \neq -1$$
$$x \neq -0.5$$

1. State any restrictions on the variables for each of the following rational expressions:

a. $\frac{3}{x+2}$ **b.** $\frac{5y}{y^2-1}$ **c.** $\frac{a+b}{a-b}$

Compare your responses with those in the Appendix, Section 2: Activity 1.

Mathematicians usually write fractions in **simplest form**. A fraction is said to be simplified or **reduced to lowest terms** if the numerator and denominator have no common factors other than 1.

Mathematical Process

- Communication
- [x] Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization

One way to simplify a fraction is to write the numerator and denominator as a product of prime factors and then cancel any common factors. For example,

$$\frac{6}{8} = \frac{\overset{1}{\cancel{2}} \times 3}{\underset{1}{\cancel{2}} \times 2 \times 2}$$

$$= \frac{3}{4}$$

This method is also used to simplify rational expressions.

Example

Simplify the expression $\dfrac{2x+4}{x^2+5x+6}$.

Solution

Factor the numerator and the denominator; then cancel any common factors.

$$\frac{2x+4}{x^2+5x+6} = \frac{2\overset{1}{\cancel{(x+2)}}}{\underset{1}{\cancel{(x+2)}}(x+3)}$$

$$= \frac{2}{x+3}$$

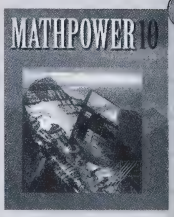
Note: Factoring the denominator of a rational expression helps you identify any non-permissible values of the variable. These values are determined from the original expression. For example, the non-permissible values of the preceding expression are found as follows:

$x+2 \neq 0$ $\qquad$ $x+3 \neq 0$

$x \neq -2$ $\qquad$ $x \neq -3$

Therefore, $x \neq -2, 3$.

For more instruction on how to simplify rational expressions, turn to page 157 of *MATHPOWER™ 10* and work through Examples 1 and 2.



2. Answer the following questions on pages 158 of the textbook. For each question, state any restrictions on the variables.

a. questions 9, 11, 13, 15, and 17 of "Practice"
b. questions 19, 21, 23, 25, and 27 of "Practice"
c. questions 29, 31, 33, 35, and 37 of "Practice"
d. questions 39, 41, 43, 45, and 47 of "Practice"

Compare your responses with those in the Appendix, Section 2: Activity 1.

Mathematical Process
- Communication
- [x] Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization

When you simplify rational numbers of the form $\frac{a}{b}$, ensure that the numerator and denominator have no common factors other than 1 and that the denominator is a natural number. Remember, too, that a negative fraction may be written three different ways:

$$-\frac{5}{8} \text{ or } \frac{-5}{8} \text{ or } \frac{5}{-8}$$

Usually, the fraction is written with a positive denominator. Similarly, when you simplify rational expressions with a monomial denominator, ensure that the numerator and denominator have no common factors other than 1 and that the denominator is positive.

Example

Rewrite the following rational expressions so the denominator has a positive coefficient:

a. $\frac{12m}{-5t}$ **b.** $\frac{6x^2-1}{-4x}$

Solution

Multiply the numerator and denominator by -1. This does not change the value of the expression.

a.
$$\frac{12m}{-5t} = \frac{-1(12m)}{-1(5t)}$$
$$= \frac{-12m}{5t}$$

Restriction: $t \neq 0$

b.
$$\frac{6x^2-1}{-4x} = \frac{-1\left(6x^2-1\right)}{-1(-4x)}$$
$$= \frac{-6x^2+1}{4x}$$
$$= \frac{1-6x^2}{4x}$$

Restriction: $x \neq 0$

3. Simplify each of the following:

a. $\frac{8a}{-3}$ **b.** $\frac{5x^2y}{-6x}$ **c.** $\frac{x-1}{-5}$ **d.** $\frac{a^2-a}{-3a}$

Compare your responses with those in the Appendix, Section 2: Activity 1.

When you are simplifying rational expressions, it is helpful to write all the terms in the numerator and denominator in either descending order or ascending order. Also, be sure to note the non-permissible value(s) of the variable(s) in each step. This will help prevent you from missing any restrictions.

Example

Simplify each of the following rational expressions:

a. $\frac{x^2+5x+6}{3x+x^2}$ **b.** $\frac{1-x}{x^2-x}$

Solution

a.
$$\frac{x^2+5x+6}{3x+x^2}=\frac{x^2+5x+6}{x^2+3x}$$
$$=\frac{(x+3)(x+2)}{x(x+3)}$$
$$=\frac{x+2}{x}$$

Restrictions: $x\neq -3,\ 0$

b.
$$\frac{1-x}{x^2-x}=\frac{-x+1}{x^2-x}$$
$$=\frac{-1(x-1)}{x(x-1)}$$
$$=\frac{-1}{x}$$

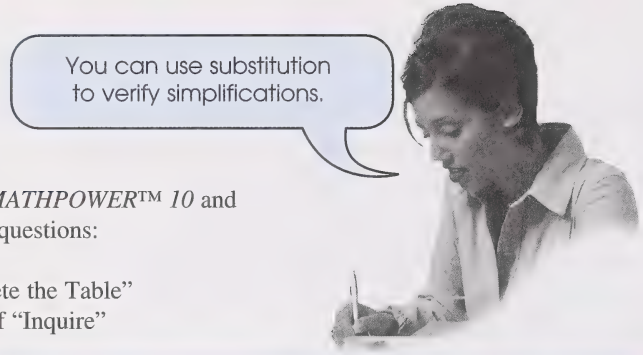
Restrictions: $x\neq 0,\ 1$

4. Simplify each of the following rational expressions. State any restrictions on the variables.

a. $\frac{2x^2+10x}{20+4x}$ **b.** $\frac{3x-12y}{20y-5x}$ **c.** $\frac{25-a^2}{2a^2-9a-5}$ **d.** $\frac{m-5}{10-2m}$

Compare your responses with those in the Appendix, Section 2: Activity 1.

Verifying Simplifications

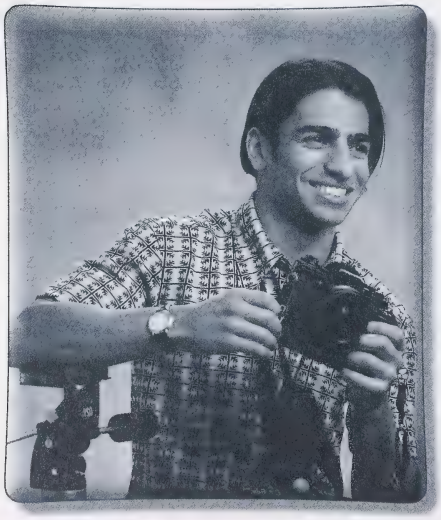


5. Turn to page 156 of *MATHPOWER™ 10* and answer the following questions:

a. "Explore: Complete the Table"
b. questions 1 to 3 of "Inquire"

Check your responses with those in the Appendix, Section 2: Activity 1.

Now Try This



Do you enjoy photography? If so, do you prefer photographing people, scenery, or animals?

Lighting is one of the conditions a photographer must consider when shooting. The size of the aperture and the shutter speed are two of the variables that affect how much light strikes the film in a camera.

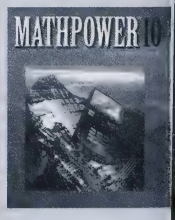
The aperture is the circular opening in the lens of a camera. When the aperture is open, light passes through and strikes the film. The size of the aperture is indicated by an f-stop. The shutter speed is the length of time the aperture is open.

6. Turn to "CAREER CONNECTION" on page 181 of *MATHPOWER™ 10* and answer questions 1 and 2 of Investigation 1, "Shutter Speed."

Compare your responses with those in the Appendix, Section 2: Activity 1.

Mathematical Process

- Communication
- [x] Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization



One of the world's most famous portrait photographers is Yousuf Karsh of Ottawa. Karsh's well-known photograph of Winston Churchill is shown on page 181 of *MATHPOWER™ 10*.

If you have access to the Internet, search for more information on Karsh. The following websites may be helpful:

http://www.mfa.org/exhibits/karsh.html
http://here-ye.com/here-ye/masters/karsh.htm

Mathematical Process

- [x] Communication
- [ ] Connection
- [ ] Estimation
- [ ] Mental Math
- [ ] Problem Solving
- [ ] Reasoning
- [ ] Technology
- [ ] Visualization

Looking Back

In this activity you simplified rational expressions. In your journal, write a letter to a friend explaining how to simplify a rational expression.

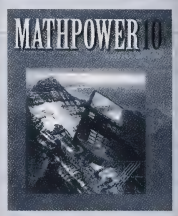
Activity 2: Multiplying and Dividing Rational Expressions

All wind instruments have much in common with each other. Therefore, once people know how to play one kind of wind instrument, they usually find it easy to learn how to play another type of wind instrument.

Similarly, once you know how to work with rational numbers, you can apply what you know to rational expressions.

Multiplying and Dividing Rational Expressions Involving Monomials

To discover how to multiply rational expressions involving monomials, turn to page 160 of *MATHPOWER™ 10* and read from the red line to the end of Example 1 on page 161. Work through the examples given. Notice that the steps used to multiply rational numbers are also used to multiply rational expressions.



Mathematical Process

- Communication
- [X] Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization

In Example 1, the numerators were multiplied, the denominators were multiplied, and then the resulting rational expression was factored and simplified. You may find it easier to factor the numerators and denominators before multiplying. Also, be sure to note the non-permissible values of the variables in each step. This will help prevent you from missing any restrictions. For example,

$$\frac{2}{3}\times\frac{-1}{4}=\frac{2\times(-1)}{3\times(4)}$$

$$=\frac{\overset{1}{\cancel{2}}\times(-1)}{3\times\underset{1}{\cancel{(2)}}(2)}$$

$$=\frac{-1}{6}\text{ or }-\frac{1}{6}$$

$$\frac{2x^2}{3}\times\frac{-1}{x}=\frac{2x^2\times(-1)}{3\times x}$$

$$=\frac{2\overset{1}{\cancel{(x)}}(x)\times(-1)}{3\times\underset{1}{\cancel{x}}}$$

$$=\frac{-2x}{3}\text{ or }-\frac{2x}{3}$$

Restriction: $x\neq 0$

Rational expressions are divided in the same way as rational numbers. For example,

$$\frac{-3}{4}\div\frac{-1}{2}=\frac{-3}{4}\times\frac{2}{-1}$$

$$=\frac{-3\times 2}{4\times(-1)}$$

$$=\frac{\overset{1}{\cancel{(-1)}}(3)\times\overset{1}{\cancel{2}}}{\underset{1}{\cancel{(2)}}(2)\times\underset{1}{\cancel{(-1)}}}$$

$$=\frac{3}{2}$$

$$\frac{-3}{x}\div\frac{2}{x^2y}=\frac{-3}{x}\times\frac{x^2y}{2}$$

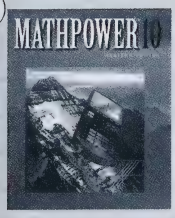
$$=\frac{-3\times\left(x^2y\right)}{x\times 2}$$

$$=\frac{(-1)(3)\times\overset{1}{\cancel{(x)}}(x)(y)}{\underset{1}{\cancel{x}}\times 2}$$

$$=\frac{-3xy}{2}\text{ or }-\frac{3xy}{2}$$

Restrictions: $x\neq 0$ and $y\neq 0$

Remember: You can check your multiplication or division by substituting 1 into the original expression and into the simplified expression. The results will be the same if the multiplication or division was performed correctly.



To discover how to multiply and divide rational expressions involving polynomials, turn to page 161 of *MATHPOWER™ 10* and work through Examples 2 and 3.

1. Answer the following questions on page 163 of the textbook. State any restrictions on the variables.

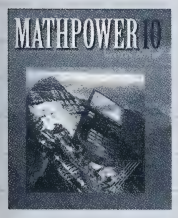
   a. questions 23, 25, and 27 of "Practice"
   b. questions 29, 31, 33, and 35 of "Practice"
   c. questions 37, 39, and 41 of "Practice"
   d. questions 43, 45, and 47 of "Practice"

2. Answer question 55 of "Applications and Problem Solving" on page 164 of the textbook.

Compare your responses with those in the Appendix, Section 2: Activity 2.

Mathematical Process

- Communication
- [x] Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization



Now Try This

Francine is a truck driver. Since time is always important when transporting any load (for example, perishable goods, merchandise, or material), Francine tries to take the route that takes the least time.

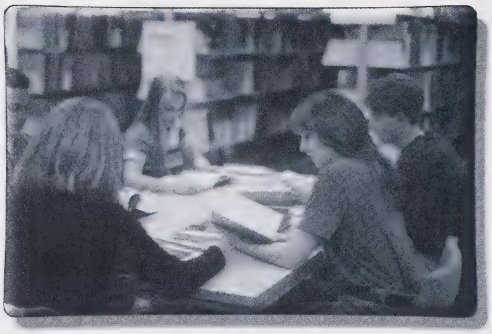
Turn to "CONNECTING MATH AND TRANSPORTATION" on page 196 of *MATHPOWER™ 10* and read "Planning Driving Routes."

3. Answer the following questions on page 197 of the textbook:

   a. questions 1 and 2 of Investigation 1, "Writing Directions"
   b. questions 1 and 2 of Investigation 2, "Comparing Routes"

Compare your responses with those in the Appendix, Section 2: Activity 2.

Finding the route that takes the least time is an optimization problem. To learn more about optimization problems, turn to "PROBLEM SOLVING" on pages 122 and 123 of *MATHPOWER™ 10* and read "Work Backward." Work through the given example carefully.



4. Answer question 1 of "Applications and Problem Solving" on page 123 of the textbook.

Compare your responses with those in the Appendix, Section 2: Activity 2.

Looking Back

In this activity, you multiplied and divided rational expressions. You also investigated problems involving driving routes.

In your journal, explain why it is important to know how to factor algebraic expressions when you multiply and divide rational expressions.

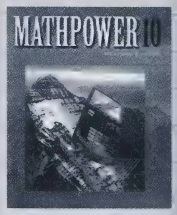
Activity 3: Adding and Subtracting Rational Expressions

Did you know that the rate at which crickets chirp depends on the temperature? The hotter it is, the more often a cricket chirps. The following formula shows how the temperature is related to the number of cricket chirps per minute:

$$T = \frac{n}{8} + 5$$

In this formula, T is the temperature (in °C) and n is the number of chirps per minute.

Mathematical Process
- Communication
- Connection
- Estimation
- Mental Math
- [x] Problem Solving
- Reasoning
- Technology
- Visualization



Mathematical Process
- [x] Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization

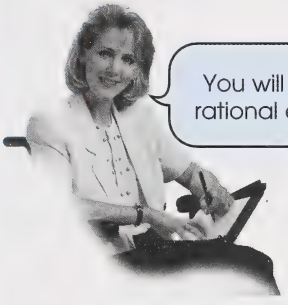
For example, if you counted 60 chirps in one minute, what would be the temperature?

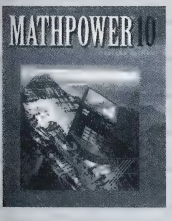
$$T = \frac{n}{8} + 5 \qquad \textbf{or} \qquad T = \frac{n}{8} + 5$$
$$= \frac{60}{8} + 5 \qquad\qquad = \frac{60}{8} + 5$$
$$= \frac{60}{8} + \frac{40}{8} \qquad\qquad = 7.5 + 5$$
$$= \frac{100}{8} \qquad\qquad = 12.5$$
$$= 12.5$$

The temperature would be about 12.5°C.

To solve this problem, it was necessary to add rational numbers. In this activity, you will examine how to add rational expressions.

Adding and Subtracting Rational Expressions with Like Denominators





To discover how adding and subtracting rational expressions is similar to adding and subtracting rational numbers, turn to page 166 of *MATHPOWER™ 10* and read from the top of the page to the red line. Work through Example 1 carefully.

Be careful when you are subtracting rational expressions with polynomial numerators. You may find it helpful to actually show the step where you add the opposite of the polynomial being subtracted.

For example, $\frac{4x-1}{x+2} - \frac{x+3}{x+2} = \frac{(4x-1)-(x+3)}{x+2}$

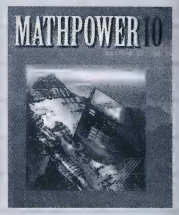
$$= \frac{(4x-1)+(-x-3)}{x+2}$$
$$= \frac{4x-1-x-3}{x+2}$$
$$= \frac{3x-4}{x+2} \qquad \textbf{Restriction: } x \neq -2$$

Remember: *Note the restrictions on the variable in each step; this will help prevent you from missing any restrictions.*

Remember: You can check your addition or subtraction by substituting 1 into the original expression and in the simplified expression. The results will be the same if the addition or subtraction was performed correctly.

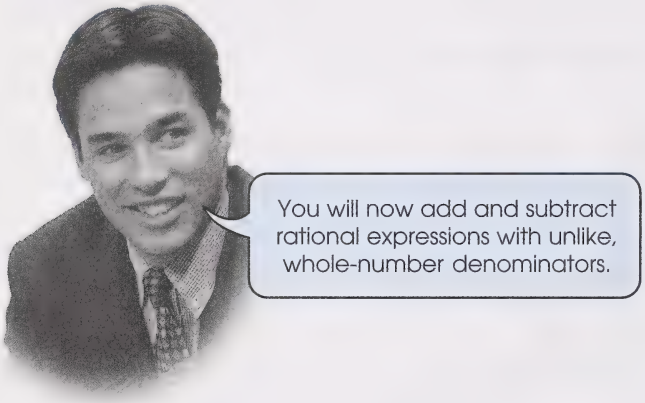
1. Answer the following questions on page 167 of *MATHPOWER™ 10*. Remember to state any restrictions on the variables.

   a. questions 1 and 3 of "Practice"
   b. questions 5, 7, and 9 of "Practice"

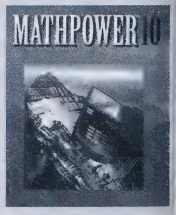


Compare your responses with those in the Appendix, Section 2: Activity 3.

Adding and Subtracting Rational Expressions with Unlike, Whole-Number Denominators



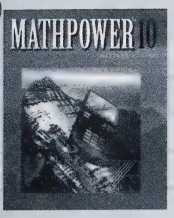
Turn to page 166 of *MATHPOWER™ 10* and read from the red line to the red line on page 167. Work through Example 2 carefully.



In Example 2, the lowest common denominator (LCD) was found by writing the prime factorization of each denominator. To find the LCD, you may find it helpful to align the common factors in columns, take one factor from each column, and multiply.

$$\begin{aligned} 4 &= 2 \times 2 \\ 8 &= 2 \times 2 \times 2 \\ 6 &= 2 \quad\quad \times 3 \end{aligned}$$

The LCD is $2 \times 2 \times 2 \times 3 = 24$.

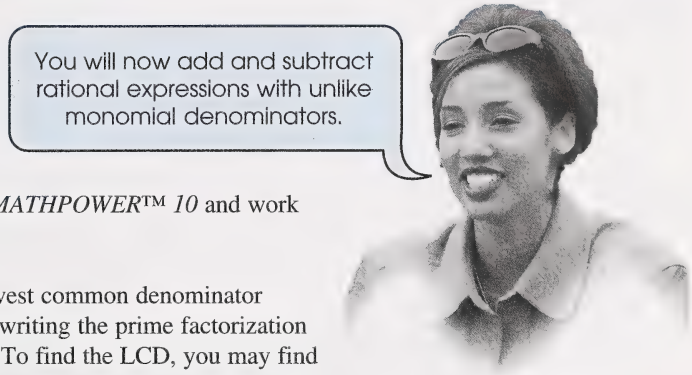


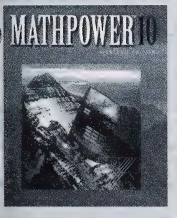
2. Answer the following questions on pages 167 and 168 of the textbook. Remember to state any restrictions on the variables.

a. questions 15 and 17 of "Practice"
b. questions 19, 21, 23, and 25 of "Practice"
c. question 29 of "Applications and Problem Solving"

Compare your responses with those in the Appendix, Section 2: Activity 3.

Adding and Subtracting Rational Expressions with Unlike, Monomial Denominators



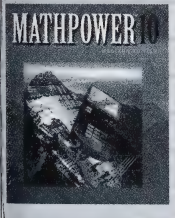


Turn to page 170 of *MATHPOWER™ 10* and work through Example 1.

In Example 1, the lowest common denominator (LCD) was found by writing the prime factorization of each denominator. To find the LCD, you may find it helpful to align the common factors in columns, take one factor from each column, and multiply.

$$\begin{aligned} 5a &= 5 \times a \\ 2a^2 &= 2 \times a \times a \\ a^3 &= a \times a \times a \end{aligned}$$

The LCD is $5 \times 2 \times a \times a \times a = 10a^3$.



3. Answer questions 9, 11, 13, and 15 of "Practice" on page 172 of the textbook. State any restrictions on the variables.

Compare your responses with those in the Appendix, Section 2: Activity 3.

Adding and Subtracting Rational Expressions with a Common Binomial Factor in the Denominators

Turn to page 170 of *MATHPOWER™ 10* and work through Example 2.

In Example 2, the lowest common denominator (LCD) was found by writing the prime factorization of each denominator. To find the LCD, you may find it helpful to align the common factors in columns, take one factor from each column, and multiply.

$$2m-4 = 2 \times (m-2)$$
$$3m-6 = 3 \times (m-2)$$

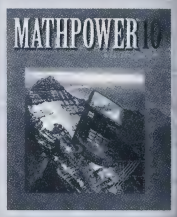
The LCD is $2(3)(m-2) = 6(m-2)$.

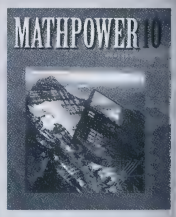
4. Answer questions 21, 23, and 25 of "Practice" on page 172 of the textbook. State any restrictions on the variables.

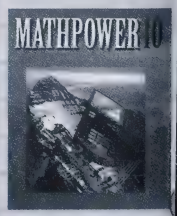
Compare your responses with those in the Appendix, Section 2: Activity 3.

Adding and Subtracting Rational Expressions with Different Binomial Denominators

Turn to page 171 of *MATHPOWER™ 10* and work through Example 3.



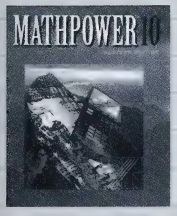




In Example 3, the lowest common denominator (LCD) was found by writing the prime factorization of each denominator. To find the LCD, you may find it helpful to align the common factors in columns, take one factor from each column, and multiply.

$$6x+6=6(x+1)= 2 \times 3 \times (x+1)$$
$$4x-12=4(x-3)= 2 \times 2 \times (x-3)$$

The LCD is $2(2)(3)(x+1)(x-3)=12(x+1)(x-3)$.

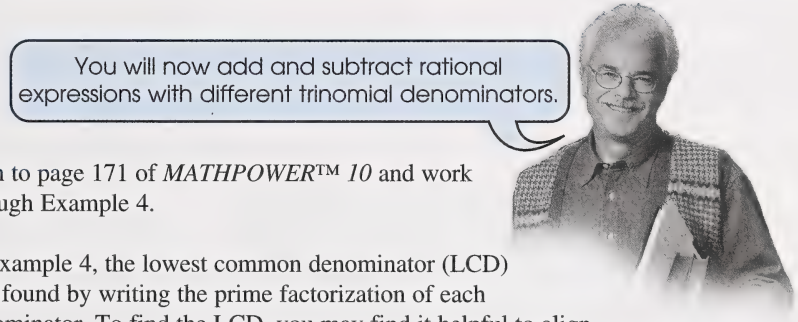


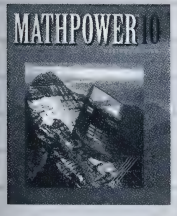
5. Answer the following questions on page 172 of the textbook. State any restrictions on the variables.

a. questions 27, 29, and 31 of "Practice"
b. questions 33 and 35 of "Practice"

Compare your responses with those in the Appendix, Section 2: Activity 3.

Adding and Subtracting Rational Expressions with Different Trinomial Denominators





Turn to page 171 of *MATHPOWER™ 10* and work through Example 4.

In Example 4, the lowest common denominator (LCD) was found by writing the prime factorization of each denominator. To find the LCD, you may find it helpful to align the common factors in columns, take one factor from each column, and multiply.

$$y^2+5y+6= (y+2) \times (y+3)$$
$$y^2-y-12= (y+3) \times (y-4)$$

The LCD is $(y+2)(y+3)(y-4)$.

6. Answer the following questions on pages 172 and 173 of the textbook. Remember to state any restrictions on the variables.

   a. questions 43, 45, and 47 of "Practice"
   b. questions 49, 51, and 53 of "Practice"
   c. questions 55, 57, 59, and 61 of "Practice"

Compare your responses with those in the Appendix, Section 2: Activity 3.

Now Try This

Throughout this course, you will have opportunities to practise every math process.

7. Turn to page 147 of *MATHPOWER™ 10* and answer questions 1 to 6 of "Mental Math: Adding Using Compatible Numbers."

8. Turn to page 173 of *MATHPOWER™ 10* and answer questions 1 and 2 of "LOGIC POWER."

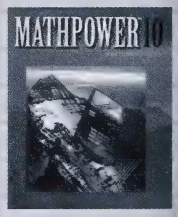
Compare your responses with those in the Appendix, Section 2: Activity 3.

Looking Back

In this activity, you added and subtracted rational expressions. In your journal, explain how to find the LCD and how it is useful in adding and subtracting rational expressions.

Mathematical Process

- [ ] Communication
- [ ] Connection
- [ ] Estimation
- [x] Mental Math
- [ ] Problem Solving
- [x] Reasoning
- [ ] Technology
- [x] Visualization



Mathematical Process

- [x] Communication
- [ ] Connection
- [ ] Estimation
- [ ] Mental Math
- [ ] Problem Solving
- [ ] Reasoning
- [ ] Technology
- [ ] Visualization

Activity 4: Rational Equations

Mathematical Process
- Communication
- [X] Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization

When you take a trip by car, do you keep track of the distances and the stops you make on the way to your destination?

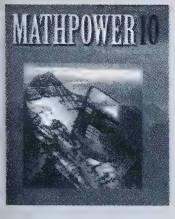
Last summer, Ruth drove 390 km from her apartment (A) in Edmonton to a cousin's farm (D) near Delmas, Saskatchewan. On her way, she stopped at Birch Lake (B) for a rest. The next stop she made was for refreshments at a cafe (C) near Marshall, Saskatchewan. It is 10 km farther from A to B than from B to C, and it is 10 km farther from B to C than C to D. How far is it from B to C?

1. a. Let the distance from B to C be x. Draw a diagram that shows A, B, C, and D, and indicate the distances between them on the diagram.

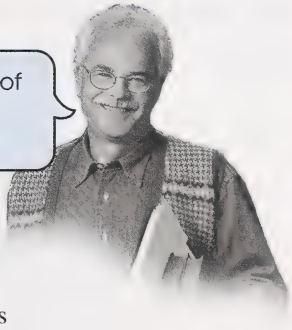
b. What is the distance from B to C? Solve the problem using the guess and check strategy.

c. Solve question 1.b. using an equation.

Compare your responses with those in the Appendix, Section 2: Activity 4.



rational equation: an equation that involves rational expressions



You have been working with equations since grade 7. By now, you are probably quite comfortable with solving equations. However, if you would like to review solving equations, you may complete pages 178 to 180 of *MATHPOWER™ 10* and compare your answers to those in the back of the textbook. If you would like to review using equations to solve problems, you may complete the questions in "Practice" on page 190 of the textbook.

In this activity, you will solve **rational equations**.

Turn to page 182 of *MATHPOWER™ 10* and read "Solving Rational Equations" and "Explore: Solve an Equation."

2. Answer questions 1 to 4 of "Inquire" on page 182 of the textbook.

Compare your responses with those in the Appendix, Section 2: Activity 4.

Solving Rational Equations with Decimals

One way to solve rational equations with decimals is to first clear the decimals by multiplying each side of the equation by 10, 100, or 1000. Work through Solution 1 of Example 1 on page 182 of *MATHPOWER™ 10*.

Another way to solve rational equations with decimals is to keep the decimals. Work through Solution 2 of Example 1 on page 183 of *MATHPOWER™ 10*.

3. Answer the following questions on page 185 of the textbook:

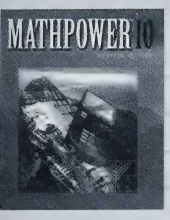
a. questions 11, 13, and 15 of "Practice"
b. questions 17 and 19 of "Practice"

Compare your responses with those in the Appendix, Section 2: Activity 4.

Solving Rational Equations with Fractions

One way to solve rational equations with fractions is to first clear the fractions by multiplying each side of the equation by the lowest common denominator (LCD). This technique is also called **clearing the denominators**.

Turn to page 184 of *MATHPOWER™ 10* and work through Examples 2, 3, and 4.



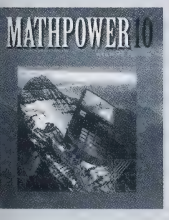
4. Answer the following questions on pages 185 and 186 of the textbook:

a. questions 5, 7, and 9 of "Practice"
b. questions 31, 33, 35, and 37 of "Practice"
c. questions 39, 41, and 43 of "Practice"
d. questions 45, 47, and 49 of "Practice"
e. questions 65, 67, and 69 of "Practice"
f. questions 70, 71, 73, 75, and 78 of "Applications and Problem Solving"

Compare your responses with those in the Appendix, Section 2: Activity 4.

Using Rational Equations to Solve Problems

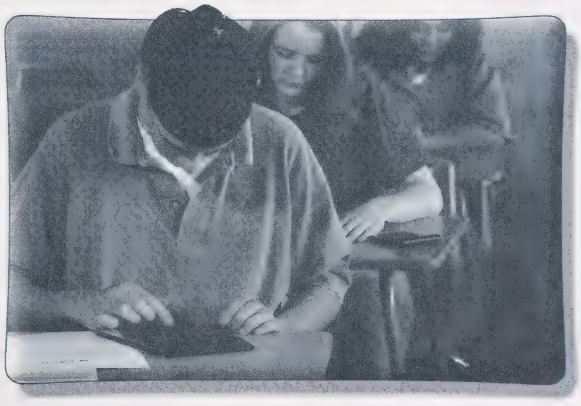
Mathematical Process
- Communication
- Connection
- Estimation
- Mental Math
- [x] Problem Solving
- Reasoning
- Technology
- Visualization



Turn to pages 188 and 189 of *MATHPOWER™ 10* and work through Examples 2, 3, and 4.

5. Answer questions 13, 14, 16, 18, 21, and 22 of "Applications and Problem Solving" on pages 190 and 191 of the textbook.

Compare your responses with those in the Appendix, Section 2: Activity 4.



Solving Equations with More Than One Variable

So far, all of the equations you have worked with have had only one variable. In this part of the activity, you will work with equations having more than one variable. You will rewrite these equations by isolating one of the variables. This process is called **solving for that variable**.

Example

The formula for the area of a triangle is $A = \frac{1}{2}bh$, where b is the base and h is the height of the triangle. Solve the formula for h, and state any non-permissible values.

Solution

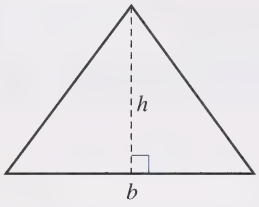
$$A = \frac{1}{2}bh$$

$$2(A) = 2\left(\frac{1}{2}\right)bh$$

$$2A = bh$$

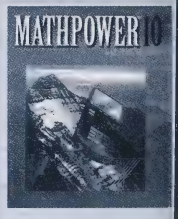
$$\frac{2A}{b} = \frac{bh}{b}$$

$$\frac{2A}{b} = h$$



$\therefore\ h = \frac{2A}{b}$ **Restriction:** $b \neq 0$

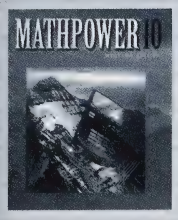
Turn to pages 192 and 193 of *MATHPOWER™ 10* and work through Examples 1 and 2.



6. Answer the following questions on pages 192 and 193 of the textbook:

- **a.** questions a. and b. of "Explore: Solve the Formula"
- **b.** questions 1 to 3 of "Inquire"
- **c.** questions 1, 3, and 5 of "Practice"
- **d.** questions 7, 9, and 11 of "Practice"
- **e.** questions 15 and 16.a. of "Applications and Problem Solving"

Compare your responses with those in the Appendix, Section 2: Activity 4.



Mathematical Process

- [x] Communication
- [ ] Connection
- [ ] Estimation
- [ ] Mental Math
- [ ] Problem Solving
- [ ] Reasoning
- [ ] Technology
- [ ] Visualization

Now Try This

7. Turn to page 203 of *MATHPOWER™ 10* and answer questions 1 and 2 of "Data Bank." **Note:** Refer to "DATA BANK" on pages 404 to 413 of the textbook.

Compare your responses with those in the Appendix, Section 2: Activity 4.

Looking Back

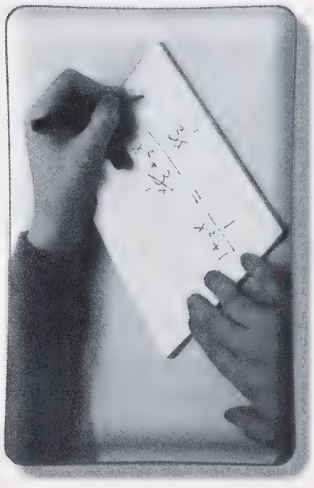
In this activity, you worked with rational equations. In your journal, explain how you can change a rational equation into an equation with integer coefficients. Describe any difficulties you had writing equations to solve problems.

Follow-up Activities

If you had difficulties understanding the concepts and skills in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts and skills, it is recommended that you do the Enrichment. You may decide to do both.

Extra Help

Todd was asked to simplify the expression $\frac{3x}{x^2+3x}$.



Todd forgot that only common **factors** in the numerator or denominator of a rational expression can be cancelled. He made the error of cancelling common **terms** in the numerator and denominator of the rational expressions.

The correct answer to the simplification is

$$\frac{3x}{x^2+3x} = \frac{3\overset{1}{\cancel{(x)}}}{\underset{1}{\cancel{x}}(x+3)}$$

$$= \frac{3}{x+3} \qquad \textbf{Restrictions: } x \neq -3,\ 0$$

Todd made an error that is commonly made by other students. Be sure that you do not make the same error.

Turn to page 200 of *MATHPOWER™ 10* and answer questions 9, 11, 13, and 15 of "Review."

Compare your responses with those in the Appendix, Section 2: Follow-up Activities, Extra Help.

Enrichment

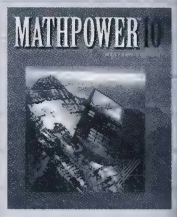
Simplifying **complex fractions** requires many of the skills you have developed in this section.

Turn to page 162 of *MATHPOWER™ 10* and read from the red line to the bottom of the page. Work through Example 4 carefully.

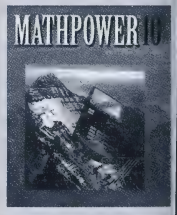
1. Answer questions 49, 51, and 53 of "Practice" on page 164 of the textbook. State any restrictions on the variables.

2. Answer question 66 of "Applications and Problem Solving" on page 173 of the textbook. State any restrictions on the variables.

Compare your responses with those in the Appendix, Section 2: Follow-up Activities, Enrichment.



complex fraction: a rational expression that contains a fraction in its numerator and/or denominator



Conclusion

In this section, you defined rational expressions, identified the non-permissible values of their variables, and operated on rational expressions. You added, subtracted, multiplied, and divided rational expressions; and you used rational expressions in equations to solve real-world problems. You discovered that rational expressions include fractions or rational numbers as a subset.

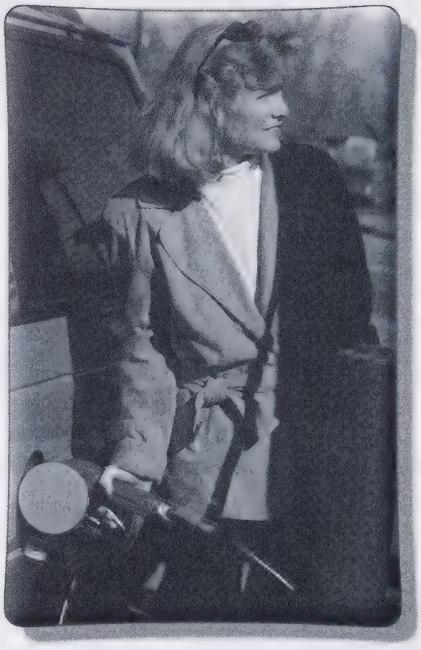
Fractions are an essential part of mathematics and they are an indispensable tool that helps you make sense of your world. Have you thought about how the gradient (or slope) of a river changes as it flows from its headwaters to the sea? The gradient increases rapidly as you move towards the headwaters of the river. The gradient or slope is represented by fractions and can be modelled by a function containing rational expressions.

Can you think of any other real-world example that can be modelled by a function containing rational expressions?

Assignment

Turn to Assignment Booklet 3B and complete the assignment for Section 2.

Module Summary



In this module, you used algebra. In Section 1, you explored polynomials. You used a calculator and paper and pencil to solve problems involving polynomials. You used algebra tiles or diagrams to help visualize polynomials. In Section 2, you solved rational equations and stated any restrictions on the variables. You also solved problems involving rational expressions and equations.

Did you know that there are sites on the Internet where you can discuss questions you have about algebra? If you have access to the Internet and you have some questions about algebra, go to the following website:

http://www.algebra-online.com

Using algebra is like learning to operate a motor vehicle. Confidence increases with instruction and practice; each new skill is built on your previous knowledge; visualization and diagrams help you overcome difficulties; and you practise your skills under the supervision of an experienced instructor.

You can relate many of your skills in algebra to driving. You can solve distance problems and gas-consumption problems. If you took driver training, your instructor probably talked to you about braking distances. Problems involving gas consumption or braking distance can be calculated using a rational equation. Can you use your skills in algebra to draw a graph showing gas consumption or braking distance?

Final Module Assignment

Turn to Assignment Booklet 3B and complete the final module assignment.

Glossary
Suggested Answers
Cut-out Learning Aids

Glossary

ascending order (in a polynomial): an arrangement of terms so that the exponents of one variable are in order from least to greatest

common factor: a number or expression that is common to two or more numbers or polynomials

complex fraction: a rational expression that contains a fraction in its numerator and/or denominator

conjugate binomials: two binomials of the form $(a+b)$ and $(a-b)$

descending order (in a polynomial): an arrangement of terms so that the exponents of one variable are in order from greatest to least

difference of squares: a binomial of the form a^2-b^2

expand: change the factor form of an expression to a polynomial

factoring over the integers: finding factors with integral coefficients when factoring polynomials with integral coefficients

greatest common factor (GCF): the greatest number or expression that is common to two or more numbers or polynomials

monomial: a number, a variable, or a product of a number and one or more variables

perfect square trinomial: a trinomial of the form $a^2+2ab+b^2$

polynomial: an algebraic expression formed by adding or subtracting monomials

prime number: a number with only two factors: 1 and itself

rational equation: an equation that involves rational expressions

rational expression: a fraction with a variable in the numerator and/or denominator

simplify: to perform the required operations and combine like terms

synthetic division: a shortcut for dividing polynomials when the divisor is $x-b$

Suggested Answers

Section 1: Activity 1

1. a. Textbook question 1 of Investigation 1, "Representing Expressions With Tiles," p. 98

1. a. Step 1: Group the like tiles.

Step 2: Write the expression represented by the tiles.

$$2x^2+3x+4$$

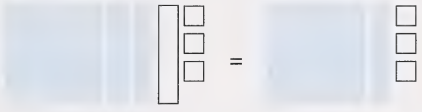
b. y^2+2y-3

c. $x^2+xy+3y$

d. $2x^2-2x+3y$

b. Textbook question 1 of Investigation 2, "The Zero Principle," p. 98

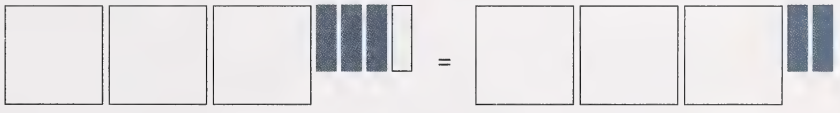
1. a. Step 1: Group the like tiles, and remove any zero pairs.



Step 2: Determine the number of tiles that must be added to the remaining tiles to make zero.

Five tiles must be added: one $-x^2$-tile, one $-x$-tile, and three 1-tiles.

b. Step 1: Group the like tiles, and remove any zero pairs.



Step 2: Determine the number of tiles that must be added to the remaining tiles to make zero.

Five tiles must be added: three x^2-tiles and two $-y$-tiles.

c. Step 1: Group the like tiles.

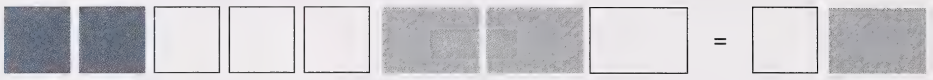
Note: There are no zero pairs to remove.

Step 2: Determine the number of tiles that must be added to the tiles to make zero.

Ten tiles must be added: two x-tiles, three $-y$-tiles, and five 1-tiles.

Section 1: Activity 1 (continued)

d. Step 1: Group the like tiles and remove any zero pairs.



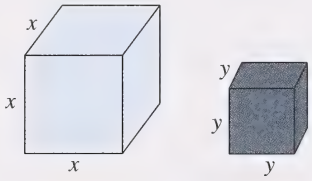
Step 2: Determine the number of tiles that must be added to the remaining tiles to make zero.

Two tiles must be added: one y^2-tile and one $-xy$-tile.

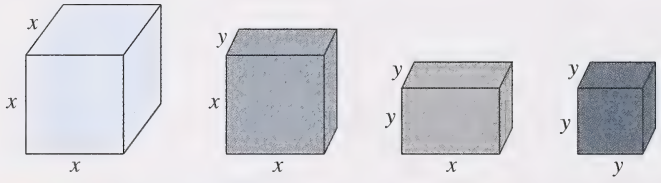
c. Textbook questions 1 and 2 of Investigation 4, "Designing Algebra Tiles," p. 99

1. A cube would be needed to model x^3.

2. a. A sketch of the shapes needed to model $x^3 + y^3$ is as follows:



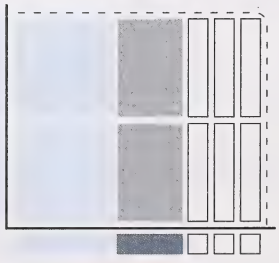
b. A sketch of the shapes needed to model $x^3 + x^2y + xy^2 + y^3$ is as follows:



2. a.

The product of $3(x+5)$ is $3x+15$.

b.



The product of $2x(x+y-3)$ is $2x^2+2xy-6x$.

3. a. Textbook questions 1, 3, 5, and 9 of "Practice," p. 107

1. $2x(3x-4)=2x(3x-4)$
$=6x^2-8x$

3. $-4t(5s-t)=-4t(5s-t)$
$=-20st+4t^2$

5. $2y^2(3y-1)=2y^2(3y-1)$
$=6y^3-2y^2$

9. $(1-6y)(-3)=-3(1-6y)$
$=-3+18y$
$=18y-3$

b. Textbook questions 17, 19, 21, 23, and 25 of "Practice," p. 107

17. $6(3a-4b-9)=6(3a-4b-9)$
$=18a-24b-54$

19. $-(4m^2-m-7)=-1(4m^2-m-7)$
$=-4m^2+m+7$
$=7+m-4m^2$

21. $-5(2x^2+3xy-y^2)=-5(2x^2+3xy-y^2)$
$=-10x^2-15xy+5y^2$
$=5y^2-15xy-10x^2$

23. $-6x^2(3x^2-6x-9)=-6x^2(3x^2-6x-9)$
$=-18x^4+36x^3+54x^2$
$=54x^2+36x^3-18x^4$

25. $abc(3a+4b-2c)=abc(3a+4b-2c)$
$=3a^2bc+4ab^2c-2abc^2$

Section 1: Activity 1 (continued)

4. a. Textbook questions 11 to 16 of "Practice," p. 107

11. $2(x-4)+5(x+3)=2(x-4)+5(x+3)$

$=2x-8+5x+15$

$=2x+5x-8+15$

$=7x+7$

12. $3(m-3)-6(m-7)=3(m-3)-6(m-7)$

$=3m-9-6m+42$

$=3m-6m-9+42$

$=-3m+33$

$=33-3m$

13. $4(2x-7)-5(4x+9)=4(2x-7)-5(4x+9)$

$=8x-28-20x-45$

$=8x-20x-28-45$

$=-12x-73$

14. $5(3t-7)-(4t+1)=5(3t-7)-1(4t+1)$

$=15t-35-4t-1$

$=15t-4t-35-1$

$=11t-36$

15. $4x+3(2x-5)+6(1-5x)=4x+3(2x-5)+6(1-5x)$

$=4x+6x-15+6-30x$

$=4x+6x-30x-15+6$

$=-20x-9$

16. $8(1-3y)-4+2(8y-7)=8(1-3y)-4+2(8y-7)$

$=8-24y-4+16y-14$

$=-24y+16y+8-4-14$

$=-8y-10$

b. Textbook questions 26, 28, and 30 of "Practice," p. 107

26. $3x(x-4)-x(x+5)-2x(x-1)=3x(x-4)-x(x+5)-2x(x-1)$

$=3x^2-12x-x^2-5x-2x^2+2x$

$=3x^2-x^2-2x^2-12x-5x+2x$

$=-15x$

28. $2x\left(x^2-3x-4\right)-3x\left(4x^2-x+5\right)=2x\left(x^2-3x-4\right)-3x\left(4x^2-x+5\right)$

$=2x^3-6x^2-8x-12x^3+3x^2-15x$

$=2x^3-12x^3-6x^2+3x^2-8x-15x$

$=-10x^3-3x^2-23x$

30. $3s\left(2s^2+st-t^2\right)-t\left(4s^2-st+3t^2\right)=3s\left(2s^2+st-t^2\right)-t\left(4s^2-st+3t^2\right)$

$=6s^3+3s^2t-3st^2-4s^2t+st^2-3t^3$

$=6s^3+3s^2t-4s^2t-3st^2+st^2-3t^3$

$=6s^3-s^2t-2st^2-3t^3$

5. a. Textbook questions a. and b. of "Explore: Use Algebra Tiles," p. 106

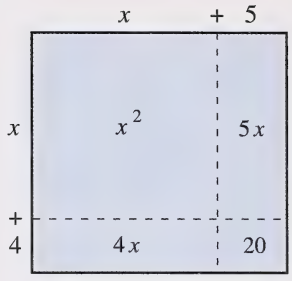
a. The area of the rectangle is x^2+5x+6.

b. $(x+3)(x+2)=\boldsymbol{x^2+5x+6}$

Section 1: Activity 1 (continued)

b. Textbook questions 31 and 44 of "Practice," p. 108

31. a. Each factor of the product $(x+4)(x+5)$ represents a dimension of a rectangle. Each term of the factor is assigned a portion of the dimension. The product of the binomials results in the sectioning of the original rectangle into four rectangular subsections.



The area of each rectangular subsection is equal to the product of its length and width.

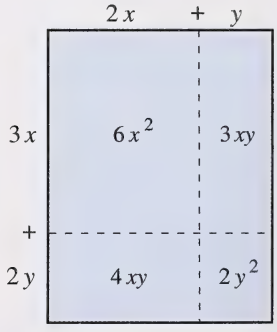
$$(x)(x) = x^2$$
$$(x)(5) = 5x$$
$$(4)(x) = 4x$$
$$(4)(5) = 20$$

The total area of the rectangle is the sum of the areas of the subsections.

b. $(x+4)(x+5) = x^2 + 5x + 4x + 20$

$$= x^2 + 9x + 20$$

44. a. Each factor of the product $(2x+y)(3x+2y)$ represents the dimensions of a rectangle. Each term of the factor is assigned a portion of the dimension. The product of the binomials results in the sectioning of the original rectangle into four rectangular subsections.



The area of each rectangular subsection is equal to the product of its length and width.

$$(3x)(2x) = 6x^2$$
$$(3x)(y) = 3xy$$
$$(2y)(2x) = 4xy$$
$$(2y)(y) = 2y^2$$

The total area of the rectangle is the sum of the areas of the subsections.

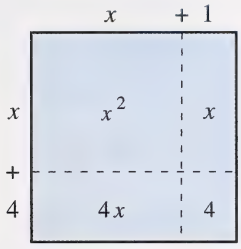
b. $(2x+y)(3x+2y) = 6x^2 + 3xy + 4xy + 2y^2$

$= 6x^2 + 7xy + 2y^2$

c. Textbook questions 1 and 5 of "Inquire," p. 106

1. a. **Algebra Tiles**

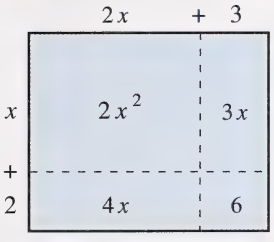
Diagram



$\therefore\ (x+4)(x+1) = x^2 + 5x + 4$

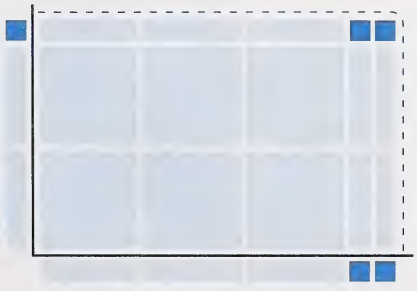
b. **Algebra Tiles**

Diagram

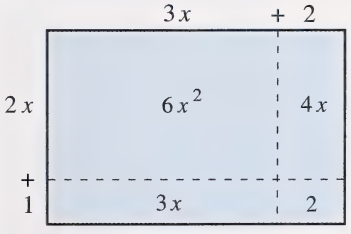


$\therefore\ (x+2)(2x+3) = 2x^2 + 7x + 6$

c. **Algebra Tiles**



Diagram

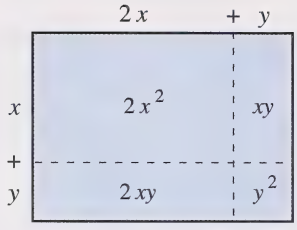


$\therefore\ (2x+1)(3x+2) = 6x^2 + 7x + 2$

Section 1: Activity 1 (continued)

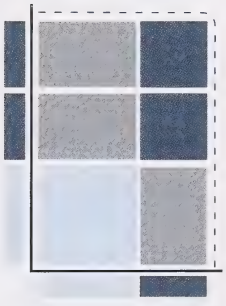
5. a. **Algebra Tiles**

Diagram

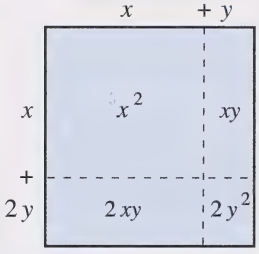


$\therefore\ (x+y)(2x+y)=2x^2+3xy+y^2$

b. **Algebra Tiles**



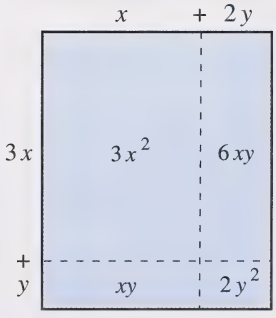
Diagram



$\therefore\ (x+2y)(x+y)=x^2+3xy+2y^2$

c. **Algebra Tiles**

Diagram



$\therefore\ (3x+y)(x+2y)=3x^2+7xy+2y^2$

6. Each factor of the product $(x+5)(x+2)$ represents a dimension of the rectangle. Each term of a factor is a part of a dimension. The product of $(x+5)$ and $(x+2)$ results in the sectioning of the original rectangle into four rectangular subsections. Each subsection has an area defined by the FOIL acronym. The total area of the rectangle is the sum of the areas of the subsections.

7. a. Textbook questions 33, 35, 37, 39, and 41 of "Practice," p. 108

33. $(m+4)(m+1) = \underbrace{m(m)}_{\text{F}} + \underbrace{m(1)}_{\text{O}} + \underbrace{4(m)}_{\text{I}} + \underbrace{4(1)}_{\text{L}}$

$= m^2 + m + 4m + 4$

$= m^2 + 5m + 4$

35. $(w-7)(w-8) = \underbrace{w(w)}_{\text{F}} + \underbrace{w(-8)}_{\text{O}} + \underbrace{(-7)(w)}_{\text{I}} + \underbrace{(-7)(-8)}_{\text{L}}$

$= w^2 - 8w - 7w + 56$

$= w^2 - 15w + 56$

37. $(y-9)(y+7) = \underbrace{y(y)}_{\text{F}} + \underbrace{y(7)}_{\text{O}} + \underbrace{(-9)(y)}_{\text{I}} + \underbrace{(-9)(7)}_{\text{L}}$

$= y^2 + 7y - 9y - 63$

$= y^2 - 2y - 63$

39. $(a-4)(a-9) = \underbrace{a(a)}_{\text{F}} + \underbrace{a(-9)}_{\text{O}} + \underbrace{(-4)(a)}_{\text{I}} + \underbrace{(-4)(-9)}_{\text{L}}$

$= a^2 - 9a - 4a + 36$

$= a^2 - 13a + 36$

41. $(2-y)(3-y) = \underbrace{2(3)}_{\text{F}} + \underbrace{2(-y)}_{\text{O}} + \underbrace{(-y)(3)}_{\text{I}} + \underbrace{(-y)(-y)}_{\text{L}}$

$= 6 - 2y - 3y + y^2$

$= 6 - 5y + y^2$

Section 1: Activity 1 (continued)

b. Textbook questions 45, 47, 49, 51, 53, and 55 of "Practice," p. 108

45. $(x+3)(3x+1) = 3x^2 + x + 9x + 3$
$= 3x^2 + 10x + 3$

47. $(y-3)(4y+5) = 4y^2 + 5y - 12y - 15$
$= 4y^2 - 7y - 15$

49. $(3x-4)(3x-4) = 9x^2 - 12x - 12x + 16$
$= 9x^2 - 24x + 16$

51. $(3a-5)(3a+5) = 9a^2 + 15a - 15a - 25$
$= 9a^2 - 25$

53. $(4a-b)(2a-5b) = 8a^2 - 20ab - 2ab + 5b^2$
$= 8a^2 - 22ab + 5b^2$

55. $(4s-3t)(5s-6t) = 20s^2 - 24st - 15st + 18t^2$
$= 20s^2 - 39st + 18t^2$

8. Textbook questions 63, 65, and 67 of "Practice," p. 108

63. $2(m+3)(m+5) + 4(2m+3) = 2\left(m^2 + 5m + 3m + 15\right) + 8m + 12$
$= 2\left(m^2 + 8m + 15\right) + 8m + 12$
$= 2m^2 + 16m + 30 + 8m + 12$
$= 2m^2 + 16m + 8m + 30 + 12$
$= 2m^2 + 24m + 42$

65. $5(3t-4)(2t-1)-(6t-5)=5\left(6t^2-3t-8t+4\right)-6t+5$

$=5\left(6t^2-11t+4\right)-6t+5$

$=30t^2-55t+20-6t+5$

$=30t^2-55t-6t+20+5$

$=30t^2-61t+25$

67. $12-2(3y-2)(3y+2)-(2y+5)(2y+5)=12-2\left(9y^2+6y-6y-4\right)-\left(4y^2+10y+10y+25\right)$

$=12-2\left(9y^2-4\right)-\left(4y^2+20y+25\right)$

$=12-18y^2+8-4y^2-20y-25$

$=-18y^2-4y^2-20y+12+8-25$

$=-22y^2-20y-5$

9. a. Textbook question "Explore: Look for Patterns," top of p. 110

Binomial Squared	Simplified Trinomial
$(x+3)^2$	x^2+6x+9
$(x-4)^2$	$x^2-8x+16$
$(2x+3)^2$	$4x^2+12x+9$
$(3x-2)^2$	$9x^2-12x+4$
$(5x+3y)^2$	$25x^2+30xy+9y^2$
$(2x-5y)^2$	$4x^2-20xy+25y^2$
$(a+b)^2$	$a^2+2ab+b^2$
$(a-b)^2$	$a^2-2ab+b^2$

b. Textbook questions 1 to 4 of "Inquire," p. 110

1. The first term in each trinomial is the square of the first term of the binomial being squared.

2. The last term of each trinomial is the square of the last term of the binomial being squared.

Section 1: Activity 1 (continued)

3. The middle term of each trinomial is twice the product of the first and last terms of the binomial being squared.

4. The square of a binomial is equal to the square of the first term, plus twice the product of the first and last terms, plus the square of the last term.

$$(a+b)^2 = (a)^2 + 2(a)(b) + (b)^2 \text{ and } (a-b)^2 = (a)^2 + 2(a)(-b) + (-b)^2$$

10. a. Textbook questions 1 to 4 of "Practice," p. 112

1. $(x+5)^2 = (x)^2 + 2(x)(5) + (5)^2$
$= x^2 + 10x + 25$

2. $(y+1)^2 = (y)^2 + 2(y)(1) + (1)^2$
$= y^2 + 2y + 1$

3. $(x-6)^2 = (x)^2 + 2(x)(-6) + (-6)^2$
$= x^2 - 12x + 36$

4. $(m-3)^2 = (m)^2 + 2(m)(-3) + (-3)^2$
$= m^2 - 6m + 9$

b. Textbook questions 10, 12, 14, and 16 of "Practice," p. 112

10. $(5x-1)^2 = (5x)^2 + 2(5x)(-1) + (-1)^2$
$= 25x^2 - 10x + 1$

12. $(2m+7)^2 = (2m)^2 + 2(2m)(7) + (7)^2$
$= 4m^2 + 28m + 49$

14. $(4y-3)^2 = (4y)^2 + 2(4y)(-3) + (-3)^2$
$= 16y^2 - 24y + 9$

16. $(2-3t)^2 = (2)^2 + 2(2)(-3t) + (-3t)^2$
$= 4 - 12t + 9t^2$

c. Textbook questions 18, 20, 22, and 24 of "Practice," p. 112

18. $(2x+3y)^2 = (2x)^2 + 2(2x)(3y) + (3y)^2$
$= 4x^2 + 12xy + 9y^2$

20. $(4t-5s)^2 = (4t)^2 + 2(4t)(-5s) + (-5s)^2$
$= 16t^2 - 40st + 25s^2$

22. $(3c+7d)^2 = (3c)^2 + 2(3c)(7d) + (7d)^2$
$= 9c^2 + 42cd + 49d^2$

24. $(a-8b)^2 = (a)^2 + 2(a)(-8b) + (-8b)^2$
$= a^2 - 16ab + 64b^2$

d. Textbook questions 54.a., 54.c., 54.e., and 54.g. of "Applications and Problem Solving," p. 113

54. a. $\left(x^2+1\right)^2 = \left(x^2\right)^2 + 2\left(x^2\right)(1) + (1)^2$
$= x^4 + 2x^2 + 1$

c. $\left(x^2+y^2\right)^2 = \left(x^2\right)^2 + 2\left(x^2\right)\left(y^2\right) + \left(y^2\right)^2$
$= x^4 + 2x^2y^2 + y^4$

e. $\left(2x^2+3\right)^2 = \left(2x^2\right)^2 + 2\left(2x^2\right)(3) + (3)^2$
$= 4x^4 + 12x^2 + 9$

g. $\left(x^2-2y^2\right)^2 = \left(x^2\right)^2 + 2\left(x^2\right)\left(-2y^2\right) + \left(-2y^2\right)^2$
$= x^4 - 4x^2y^2 + 4y^4$

Section 1: Activity 1 (continued)

11. a. Textbook question "Explore: Look for Patterns," bottom of p. 110

Multiplication	Simplified Product
$(x+4)(x-4)$	x^2-16
$(y-5)(y+5)$	y^2-25
$(2x-1)(2x+1)$	$4x^2-1$
$(3x+2y)(3x-2y)$	$9x^2-4y^2$
$(x^2-1)(x^2+1)$	x^4-1
$(a+b)(a-b)$	a^2-b^2

b. Textbook questions 1 to 5 of "Inquire," p. 111

1. The two binomials in each multiplication have the same first and last term (not including the sign).

2. The two binomials have different signs between the first and last terms.

3. The product is the difference of the squares of the first term and second term of the two binomials.

4. The products of the outside terms and the inside terms cancel.

5. The product of these types of binomials (conjugate binomials) is equal to the difference of the squares of the terms.

$$(a+b)(a-b)=(a)^2-(b)^2$$

12. a. Textbook questions 5 to 8 of "Practice," p. 112

5. $(x+3)(x-3)=(x)^2-(3)^2$
$=x^2-9$

6. $(y+6)(y-6)=(y)^2-(6)^2$
$=y^2-36$

7. $(m-7)(m+7)=(m)^2-(7)^2$
$=m^2-49$

8. $(t-8)(t+8)=(t)^2-(8)^2$
$=t^2-64$

b. Textbook questions 11, 13, and 15 of "Practice," p. 112

11. $(2x+3)(2x-3)=(2x)^2-(3)^2$
$=4x^2-9$

13. $(3y-2)(3y+2)=(3y)^2-(2)^2$
$=9y^2-4$

15. $(1+5m)(1-5m)=(1)^2-(5m)^2$
$=1-25m^2$

c. Textbook questions 17, 19, 21, and 23 of "Practice," p. 112

17. $(2x+3y)(2x-3y)=(2x)^2-(3y)^2$
$=4x^2-9y^2$

19. $(3a-b)(3a+b)=(3a)^2-(b)^2$
$=9a^2-b^2$

21. $(4m-5n)(4m+5n)=(4m)^2-(5n)^2$
$=16m^2-25n^2$

23. $(y+6x)(y-6x)=(y)^2-(6x)^2$
$=y^2-36x^2$

d. Textbook questions 55.a, 55.c., 55.e., and 55.f. of "Applications and Problem Solving," p. 113

55. a. $\left(x^2+1\right)\left(x^2-1\right)=\left(x^2\right)^2-(1)^2$
$=x^4-1$

c. $\left(x^2+y^2\right)\left(x^2-y^2\right)=\left(x^2\right)^2-\left(y^2\right)^2$
$=x^4-y^4$

e. $\left(3x^2+2y^2\right)\left(3x^2-2y^2\right)=\left(3x^2\right)^2-\left(2y^2\right)^2$
$=9x^4-4y^4$

f. $\left(4-3c^2\right)\left(4+3c^2\right)=(4)^2-\left(3c^2\right)^2$
$=16-9c^4$

13. a. Textbook questions 28 to 30 of "Practice," p. 112

28. $2(a+3)(a-3)+3(a+2)^2=2\left(a^2-9\right)+3\left(a^2+4a+4\right)$
$=2a^2-18+3a^2+12a+12$
$=2a^2+3a^2+12a-18+12$
$=5a^2+12a-6$

Section 1: Activity 1 (continued)

29. $4(2x+1)^2-2(3x-7)^2=4\left(4x^2+4x+1\right)-2\left(9x^2-42x+49\right)$

$=16x^2+16x+4-18x^2+84x-98$

$=16x^2-18x^2+16x+84x+4-98$

$=-2x^2+100x-94$

30. $5(3t-1)^2-4(4t-5)(4t+5)=5\left(9t^2-6t+1\right)-4\left(16t^2-25\right)$

$=45t^2-30t+5-64t^2+100$

$=45t^2-64t^2-30t+5+100$

$=-19t^2-30t+105$

b. Textbook quesitons 35, 37, and 39 of "Practice," p. 112

35. $12-2(3y+1)^2-(y-9)(3y+2)=12-2\left(9y^2+6y+1\right)-\left(3y^2+2y-27y-18\right)$

$=12-18y^2-12y-2-\left(3y^2-25y-18\right)$

$=12-18y^2-12y-2-3y^2+25y+18$

$=-18y^2-3y^2-12y+25y+12-2+18$

$=-21y^2+13y+28$

37. $4(5s+t)(5s-t)-6\left(3t^2-t\right)=4\left(25s^2-t^2\right)-18t^2+6t$

$=100s^2-4t^2-18t^2+6t$

$=100s^2-22t^2+6t$

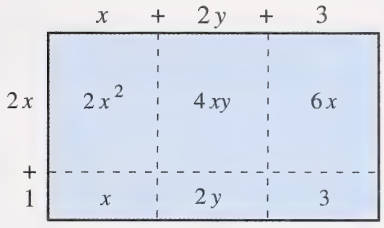
39. $(x+2y)(x-2y)+(2x+y)^2=x^2-4y^2+4x^2+4xy+y^2$

$=x^2+4x^2+4xy-4y^2+y^2$

$=5x^2+4xy-3y^2$

14. Textbook question 68 of "Practice," p. 108

68. a. Each factor of the product $(2x+1)(x+2y+3)$ represents a dimension of a rectangle. Each term of the factor is assigned a portion of the dimension. The product of the binomials results in the sectioning of the original rectangle into six rectangular subsections.



The area of each subsection is equal to the product of its length and width.

$$(2x)(x)=2x^2$$
$$(2x)(2y)=4xy$$
$$(2x)(3)=6x$$
$$(1)(x)=x$$
$$1(2y)=2y$$
$$1(3)=3$$

The total area of the rectangle is the sum of the areas of the subsections.

b. $(2x+1)(x+2y+3)=2x^2+4xy+6x+x+2y+3$

$$=2x^2+4xy+7x+2y+3$$

15. Textbook questions 69 to 72 of "Practice," p. 108

69. $(x+3)\left(x^2+2x+4\right)=x\left(x^2+2x+4\right)+3\left(x^2+2x+4\right)$

$$=x^3+2x^2+4x+3x^2+6x+12$$
$$=x^3+2x^2+3x^2+4x+6x+12$$
$$=x^3+5x^2+10x+12$$

70. $(y-2)\left(y^2-y-5\right)=y\left(y^2-y-5\right)+(-2)\left(y^2-y-5\right)$

$$=y^3-y^2-5y-2y^2+2y+10$$
$$=y^3-y^2-2y^2-5y+2y+10$$
$$=y^3-3y^2-3y+10$$

Section 1: Activity 1 (continued)

71. $(3m+2)(2m^2+3m-4) = 3m(2m^2+3m-4)+2(2m^2+3m-4)$

$= 6m^3+9m^2-12m+4m^2+6m-8$

$= 6m^3+9m^2+4m^2-12m+6m-8$

$= 6m^3+13m^2-6m-8$

72. $(t^2-5t-7)(2t+1) = 2t(t^2-5t-7)+1(t^2-5t-7)$

$= 2t^3-10t^2-14t+t^2-5t-7$

$= 2t^3-10t^2+t^2-14t-5t-7$

$= 2t^3-9t^2-19t-7$

16. a. Textbook question 42 of "Practice," p. 112

42. $(2m-1)(3m+4)^2 = (2m-1)(9m^2+24m+16)$

$= 2m(9m^2+24m+16)+(-1)(9m^2+24m+16)$

$= 18m^3+48m^2+32m-9m^2-24m-16$

$= 18m^3+48m^2-9m^2+32m-24m-16$

$= 18m^3+39m^2+8m-16$

b. Textbook questions 51.a. and 56.a. of "Applications and Problem Solving," pp. 112 and 113

51. a. $(x+3)(x-3)(x-3) = (x^2-9)(x-3)$

$= x^2(x)+x^2(-3)+(-9)(x)+(-9)(-3)$

$= x^3-3x^2-9x+27$

56. a. $(6y-1)(6y+1)(y-1)^2 = (36y^2-1)(y^2-2y+1)$

$= 36y^2(y^2-2y+1)+(-1)(y^2-2y+1)$

$= 36y^4-72y^3+36y^2-y^2+2y-1$

$= 36y^4-72y^3+35y^2+2y-1$

17. Textbook questions 43, 45, and 47 of "Practice," p. 112

43. $(x+2)^3 = (x+2)(x+2)^2$

$= (x+2)\left(x^2+4x+4\right)$

$= x\left(x^2+4x+4\right)+2\left(x^2+4x+4\right)$

$= x^3+4x^2+4x+2x^2+8x+8$

$= x^3+4x^2+2x^2+4x+8x+8$

$= x^3+6x^2+12x+8$

45. $(4x+5)^3 = (4x+5)(4x+5)^2$

$= (4x+5)\left(16x^2+40x+25\right)$

$= 4x\left(16x^2+40x+25\right)+5\left(16x^2+40x+25\right)$

$= 64x^3+160x^2+100x+80x^2+200x+125$

$= 64x^3+160x^2+80x^2+100x+200x+125$

$= 64x^3+240x^2+300x+125$

47. $(2y-x)^3 = (2y-x)(2y-x)^2$

$= (2y-x)\left(4y^2-4xy+x^2\right)$

$= 2y\left(4y^2-4xy+x^2\right)+(-x)\left(4y^2-4xy+x^2\right)$

$= 8y^3-8xy^2+2x^2y-4xy^2+4x^2y-x^3$

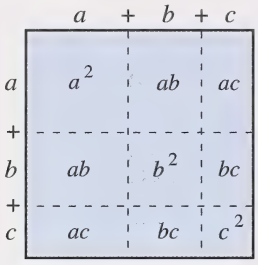
$= 8y^3-8xy^2-4xy^2+2x^2y+4x^2y-x^3$

$= 8y^3-12xy^2+6x^2y-x^3$

Section 1: Activity 1 (continued)

18. Textbook question 58.a. of "Applications and Problem Solving," p. 113

58. a. Step 1: Fill in the area of each subsection of the square.



Step 2: Find the total area of the square by adding the subsections of the square.

$$(a+b+c)(a+b+c)=a^2+b^2+c^2+2ab+2ac+2bc$$

19. Texbook questions 73 and 75 of "Practice," p. 108

73. $\left(x^2+2x-1\right)\left(x^2-x-4\right)=x^2\left(x^2-x-4\right)+2x\left(x^2-x-4\right)+(-1)\left(x^2-x-4\right)$

$$=x^4-x^3-4x^2+2x^3-2x^2-8x-x^2+x+4$$
$$=x^4-x^3+2x^3-4x^2-2x^2-x^2-8x+x+4$$
$$=x^4+x^3-7x^2-7x+4$$

75. $\left(3a^2-4a+2\right)\left(a^2-a-5\right)=3a^2\left(a^2-a-5\right)+(-4a)\left(a^2-a-5\right)+2\left(a^2-a-5\right)$

$$=3a^4-3a^3-15a^2-4a^3+4a^2+20a+2a^2-2a-10$$
$$=3a^4-3a^3-4a^3-15a^2+4a^2+2a^2+20a-2a-10$$
$$=3a^4-7a^3-9a^2+18a-10$$

20. Textbook questions 1 to 9 of "Mental Math: Multiplying Two Numbers That Differ by 2," p. 97

1. $12^2 = 144$
$144 - 1 = 143$
So, $13 \times 11 = 143$.

2. $13^2 = 169$
$169 - 1 = 168$
So, $14 \times 12 = 168$.

3. $20^2 = 400$
$400 - 1 = 399$
So, $19 \times 21 = 399$.

4. $30^2 = 900$
$900 - 1 = 899$
So, $29 \times 31 = 899$.

5. $25^2 = 625$
$625 - 1 = 624$
So, $24 \times 26 = 624$.

6. $40^2 = 1600$
$1600 - 1 = 1599$
So, $41 \times 39 = 1599$.

7. $100^2 = 10\,000$
$10\,000 - 1 = 9999$
So, $101 \times 99 = 9999$.

8. $60^2 = 3600$
$3600 - 1 = 3599$
So, $59 \times 61 = 3599$.

9. $1000^2 = 1\,000\,000$
$1\,000\,000 - 1 = 999\,999$
So, $999 \times 1001 = 999\,999$.

21. a. Textbook questions 1 to 20 of "Mental Math: Integers," p. 97

1. 2 and 2

2. 2 and 3

3. 1 and 5

4. 2 and 4

5. 3 and 4

6. 2 and 6

7. 4 and 5

8. 3 and 8

9. 4 and 6

10. 2 and 15

11. -1 and -1

12. -2 and 1

13. -5 and 1

14. -3 and 3

15. -2 and 3

16. -2 and 5

17. -7 and -1

18. -4 and -4

19. -8 and -2

20. -5 and 3

Section 1: Activity 2

1. a. $42 = 2 \times 3 \times 7$
b. $45 = 3 \times 3 \times 5$
c. $100 = 2 \times 2 \times 5 \times 5$
d. $110 = 2 \times 5 \times 11$

Section 1: Activity 2 (continued)

2. **Step 1:** Write the prime factorization of each number.

$$78 = 2 \times 3 \times 13$$
$$117 = 3 \times 3 \times 13$$

Step 2: Arrange the prime factorizations so that the common prime factors of each number are aligned in columns.

$$78 = 2 \times 3 \times 13$$
$$117 = 3 \times 3 \times 13$$

Step 3: To find the greatest common factor, take one factor from each complete column and multiply.

$$\text{GCF} = 3 \times 13$$
$$= 39$$

The most that each doorknob set could have cost is $39.

3. **Textbook question "Explore: Use Algebra Tiles," p. 118**

Rectangle	Area	Width	Length	Width × Length
A	x^2+3x	x	$x+3$	$x(x+3)$
B	$3y^2+y$	y	$3y+1$	$y(3y+1)$
C	$2x^2+6x$	$2x$	$x+3$	$2x(x+3)$
D	$2xy+2y^2$	$2y$	$x+y$	$2y(x+y)$

4. **Textbook questions 1 and 2 of "Inquire," p. 119**

1. Check the numerical coefficients for a common factor. Check the variable parts for a common factor. The product of the common factors of the numerical coefficients and the variable parts is the common factor of the polynomial. Divide each term of the polynomial by the common factor to find the remaining factor.

2. **a.** $5x^2+20x = 5x(x+4)$

b. $6x^2-3x = 3x(2x-1)$

c. $12ab+8ac-4ad = 4a(3b+2c-d)$

5. The process "factoring over the integers" means finding factors with integral coefficients.

6. a. Textbook questions 1 to 6 of "Practice," p. 120

1. $5x+25=5(x+5)$

2. $4x+13$ cannot be factored over the integers.

3. $8x+8=8(x+1)$

4. $9y-9=9(y-1)$

5. $3x-15y=3(x-5y)$

6. $25x^2+10x=5x(5x+2)$

b. Textbook questions 11, 13, 15, 17, 19, and 21 of "Practice," p. 120

11. $9a^3+27b^2=9\left(a^3+3b^2\right)$

13. $12y-8y^2+24y^3=4y\left(3-2y+6y^2\right)$

15. $6rst+3rs-7t$ cannot be factored.

17. $24xy^2+16x^2y=8xy(3y+2x)$

19. $5rst-15ab+7cd$ cannot be factored.

21. $27a^2b^3+9a^2b^2-18a^3b^2=9a^2b^2(3b+1-2a)$

7. Textbook questions 23, 25, 27, and 29 of "Practice," p. 120

23. $5x(a+b)+3(a+b)=(a+b)(5x+3)$

25. $7x(m+4)-3(m-4)$ cannot be factored.

27. $4t(m+7)+(m+7)=(m+7)(4t+1)$

29. $8x(m-n)+6(m-n)=(m-n)(8x+6)$
$=2(m-n)(4x+3)$

8. Textbook questions 31, 33, and 35 of "Practice," p. 120

31. $xy+12+4x+3y=(xy+4x)+(3y+12)$
$=x(y+4)+3(y+4)$
$=(y+4)(x+3)$

33. $m^2-4n+4m-mn=m^2+4m-mn-4n$
$=\left(m^2+4m\right)+(-mn-4n)$
$=m(m+4)-n(m+4)$
$=(m+4)(m-n)$

Section 1: Activity 2 (continued)

35. $5m^2t-10m^2+t^2-2t=\left(5m^2t-10m^2\right)+\left(t^2-2t\right)$

$=5m^2(t-2)+t(t-2)$

$=(t-2)\left(5m^2+t\right)$

9. a. Textbook questions a. and b. of "Explore: Use Algebra Tiles," p. 125

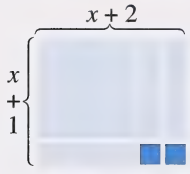
a. The length is $x+3$.

b. The width is $x+1$.

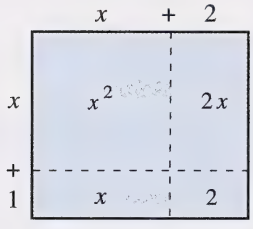
b. Textbook questions 1 to 4 of "Inquire," p. 125

1. $x^2+4x+3=(\boldsymbol{x+3})(\boldsymbol{x+1})$

2. a. **Algebra Tiles**

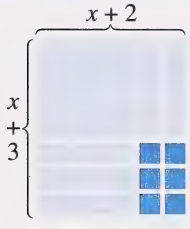


Diagram

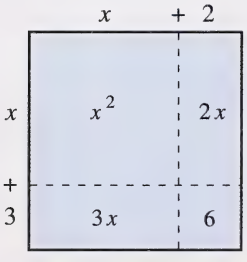


$\therefore\ x^2+3x+2=(x+1)(x+2)$

b. **Algebra Tiles**

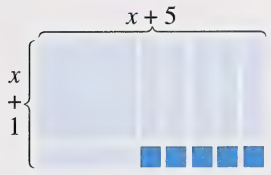


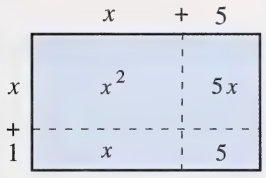
Diagram



$\therefore\ x^2+5x+6=(x+3)(x+2)$

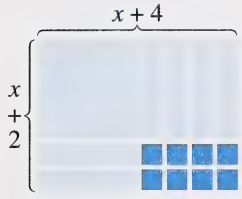
c. **Algebra Tiles** **Diagram**

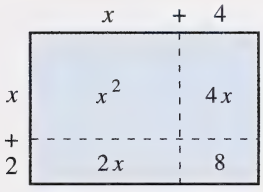




$\therefore\ x^2+6x+5=(x+1)(x+5)$

d. **Algebra Tiles** **Diagram**





$\therefore\ x^2+6x+8=(x+2)(x+4)$

3. a. The first term in the trinomial is the product of the first terms in the binomials.

b. The coefficient of the middle term in the trinomial is the sum of the second terms of the binomials.

c. The last term in the trinomial is the product of the second terms of the binomials.

4. a. $x^2+26x+25=(x+25)(x+1)$

Therefore, the length is $x+25$ and the width is $x+1$.

b. Length $= x+25$ Width $= x+1$

$= 9+25$ $= 9+1$

$= 34$ $= 10$

The length of the Jumbo Tron is 34 m and the width is 10 m.

Section 1: Activity 2 (continued)

c. Textbook question 62 of "Applications and Problem Solving," p. 127

62. You cannot arrange the algebra tiles that represent x^2+4x+5 into a rectangle. This is because the polynomial cannot be factored.

10. a. Textbook questions 9, 11, 13, 15, and 17 of "Practice," p. 126

9. The two numbers whose product is 4 and whose sum is 5 are 1 and 4.

$\therefore\ x^2+5x+4=(x+1)(x+4)$

Check: Let $x=1$.

LS	RS
x^2+5x+4	$(x+1)(x+4)$
$=1^2+5(1)+4$	$=(1+1)(1+4)$
$=1+5+4$	$=2(5)$
$=10$	$=10$

LS = RS

11. The two numbers whose product is 10 and whose sum is 7 are 2 and 5.

$\therefore\ m^2+7m+10=(m+2)(m+5)$

Check: Let $m=1$.

LS	RS
$m^2+7m+10$	$(m+2)(m+5)$
$=1^2+7(1)+10$	$=(1+2)(1+5)$
$=1+7+10$	$=3(6)$
$=18$	$=18$

LS = RS

13. The two numbers whose product is 42 and whose sum is -13 are -7 and -6.

$\therefore\ r^2-13r+42=(r-7)(r-6)$

Check: Let $r=1$.

LS	RS
$r^2-13r+42$	$(r-7)(r-6)$
$=1^2-13(1)+42$	$=(1-7)(1-6)$
$=1-13+42$	$=-6(-5)$
$=30$	$=30$

LS = RS

15. The two numbers whose product is 10 and whose sum is -7 are -5 and -2.

$\therefore\ r^2-7r+10=(r-5)(r-2)$

Check: Let $r=1$.

LS	RS
$r^2-7r+10$	$(r-5)(r-2)$
$=1^2-7(1)+10$	$=(1-5)(1-2)$
$=1-7+10$	$=-4(-1)$
$=4$	$=4$

LS = RS

17. No two numbers exist whose product is 24 and whose sum is -9; therefore, $x^2-9x+24$ cannot be factored.

b. Textbook questions 19, 21, 23, and 25 of "Practice," p. 127

19. $y^2 - y - 20 = (y-5)(y+4)$

21. $t^2 + t - 18$ cannot be factored.

23. $n^2 - 10n - 24 = (n+2)(n-12)$

25. $r^2 + 7r - 20$ cannot be factored.

c. Textbook questions 27, 29, 31, 33, 35, 37, 39, 41, 49, and 50 of "Practice," p. 127

27. $m^2 + 18m + 80 = (m+8)(m+10)$

29. $x^2 + 2x + 5$ cannot be factored.

31. $y^2 - 17y + 72 = (y-8)(y-9)$

33. $t^2 - 15t + 26 = (t-2)(t-13)$

35. $m^2 + 7m - 6$ cannot be factored.

37. $x^2 - 10x + 21 = (x-3)(x-7)$

39. $r^2 - r - 30 = (r-6)(r+5)$

41. $n^2 - 4n + 5$ cannot be factored.

49. $8 + 7y - y^2 = (8-y)(1+y)$

50. $16 - 6x - x^2 = (8+x)(2-x)$

d. Textbook questions 65 and 66 of "Applications and Problem Solving," p. 127

65. Answers will vary. Sample answers are given.

a. $(x+1)(x+1) = x^2 + 2x + 1$

$(x+3)(x-1) = x^2 + 2x - 3$

$(x+5)(x-3) = x^2 + 2x - 15$

Therefore, k can be 1, -3, or -15.

b. $(x-6)(x+1) = x^2 - 5x - 6$

$(x-7)(x+2) = x^2 - 5x - 14$

$(x-8)(x+3) = x^2 - 5x - 24$

Therefore, k can be -6, -14, or -24.

66. a. $(x+1)(x+6) = x^2 + 7x + 6$

$(x-1)(x-6) = x^2 - 7x + 6$

$(x+2)(x+3) = x^2 + 5x + 6$

$(x-2)(x-3) = x^2 - 5x + 6$

Therefore, k can be ± 7 or ± 5.

Section 1: Activity 2 (continued)

b. $(x+1)(x-12)=x^2-11x-12$ $\quad$ $(x-2)(x+6)=x^2+4x-12$

$(x-1)(x+12)=x^2+11x-12$ $\quad$ $(x+3)(x-4)=x^2-x-12$

$(x+2)(x-6)=x^2-4x-12$ $\quad$ $(x-3)(x+4)=x^2+x-12$

Therefore, k can be ± 11, ± 4, or ± 1.

c. $(x+1)(x+24)=x^2+25x+24$ $\quad$ $(x+3)(x+8)=x^2+11x+24$

$(x-1)(x-24)=x^2-25x+24$ $\quad$ $(x-3)(x-8)=x^2-11x+24$

$(x+2)(x+12)=x^2+14x+24$ $\quad$ $(x+4)(x+6)=x^2+10x+24$

$(x-2)(x-12)=x^2-14x+24$ $\quad$ $(x-4)(x-6)=x^2-10x+24$

Therefore, k can be ± 25, ± 14, ± 11, or ± 10.

d. $(x+1)(x-18)=x^2-17x-18$ $\quad$ $(x-2)(x+9)=x^2+7x-18$

$(x-1)(x+18)=x^2+17x-18$ $\quad$ $(x+3)(x-6)=x^2-3x-18$

$(x+2)(x-9)=x^2-7x-18$ $\quad$ $(x-3)(x+6)=x^2+3x-18$

Therefore, k can be ± 17, ± 7, or ± 3.

11. Textbook question 67. a. to 67. c. of "Applications and Problem Solving," p. 127

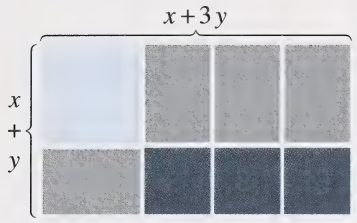
67. a. $x^4+2x^2+1=\left(x^2+1\right)\left(x^2+1\right)$

b. $x^4+x^2-6=\left(x^2+3\right)\left(x^2-2\right)$

c. $x^4-3x^2-10=\left(x^2-5\right)\left(x^2+2\right)$

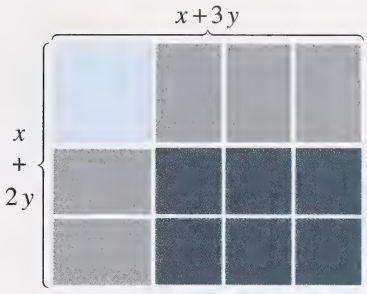
12. Textbook question 63 of "Applications and Problem Solving," p.127

63. a.



$\therefore\ x^2+4xy+3y^2=(x+y)(x+3y)$

b.



$\therefore\ x^2+5xy+6y^2=(x+2y)(x+3y)$

13. Textbook questions 43 to 47 of "Practice," p. 127

43. $a^2-4ab-77b^2=(a+7b)(a-11b)$

44. $c^2-cd-2d^2=(c+d)(c-2d)$

45. $x^2+5xy-36y^2=(x+9y)(x-4y)$

46. $x^2-4xy+6y^2$ cannot be factored.

47. $x^2y^2+3xy+2=(xy+1)(xy+2)$

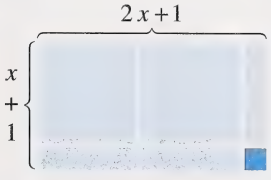
14. a. Textbook questions a. and b. of "Explore: Use Algebra Tiles," p. 128

a. The length is $2x+1$.
b. The width is $x+2$.

b. Textbook questions 1 to 3 of "Inquire," p. 128

1. $2x^2+5x+2=(\mathbf{2x+1})(\mathbf{x+2})$

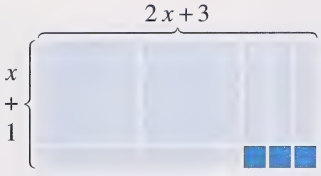
2. a.



$\therefore\ 2x^2+3x+1=(x+1)(2x+1)$

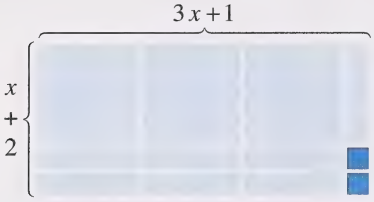
Section 1: Activity 2 (continued)

b.



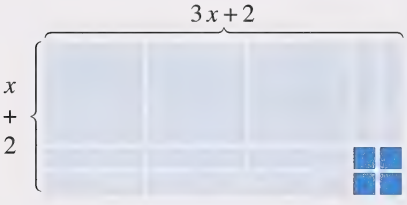
$\therefore\ 2x^2+5x+3=(x+1)(2x+3)$

c.



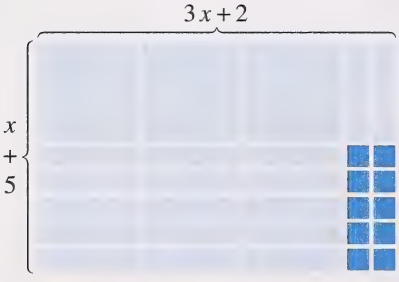
$\therefore\ 3x^2+7x+2=(x+2)(3x+1)$

d.



$\therefore\ 3x^2+8x+4=(x+2)(3x+2)$

3. a.



$\therefore\ 3x^2+17x+10=(x+5)(3x+2)$

The length of the swimming pool is $3x+2$ and the width is $x+5$.

b. Length $= 3x + 2$
$= 3(16) + 2$
$= 48 + 2$
$= 50$

Width $= x + 5$
$= 16 + 5$
$= 21$

The dimensions of the pool are 50 m by 21 m.

15. Textbook questions 51, 53, 55, 57, and 59 of "Practice," p. 127

51. $2x^2 - 6x + 4 = 2\left(x^2 - 3x + 2\right)$
$= 2(x-1)(x-2)$

53. $5y^2 + 40y + 60 = 5\left(y^2 + 8y + 12\right)$
$= 5(y+2)(y+6)$

55. $6x^2 + 18x - 24 = 6\left(x^2 + 3x - 4\right)$
$= 6(x+4)(x-1)$

57. $x^3 + 18x^2 + 72x = x\left(x^2 + 18x + 72\right)$
$= x(x+6)(x+12)$

59. $5w^2 + 20w - 60 = 5\left(w^2 + 4w - 12\right)$
$= 5(w+6)(w-2)$

16. a. Textbook questions 7, 9, and 11 of "Practice," p. 130

7. Method 1: Using Guess and Check

Possible Factors	Trinomial	Is the Guess Correct?
$(x+3)(2x+1)$	$2x^2 + 7x + 3$	No
$(x-3)(2x-1)$	$2x^2 - 7x + 3$	No
$(x-1)(2x-3)$	$2x^2 - 5x + 3$	Yes

$\therefore\ 2x^2 - 5x + 3 = (x-1)(2x-3)$

Method 2: Breaking Up the Middle Term

$2x^2 - 5x + 3 = 2x^2 - 2x - 3x + 3$
$= 2x(x-1) - 3(x-1)$
$= (x-1)(2x-3)$

Section 1: Activity 2 (continued)

9. Method 1: Using Guess and Check

Possible Factors	Trinomial	Is the Guess Correct?
$(t+4)(3t+2)$	$3t^2+14t+8$	No
$(t-4)(3t-2)$	$3t^2-14t+8$	No
$(t-2)(3t-4)$	$3t^2-10t+8$	Yes

$\therefore\ 3t^2-10t+8=(t-2)(3t-4)$

Method 2: Breaking Up the Middle Term

$$\begin{aligned}3t^2-10t+8&=3t^2-6t-4t+8\\&=3t(t-2)-4(t-2)\\&=(t-2)(3t-4)\end{aligned}$$

11. Method 1: Using Guess and Check

Possible Factors	Trinomial	Is the Guess Correct?
$(6m-6)(m-1)$	$6m^2-12m+6$	No
$(6m-2)(m-3)$	$6m^2-20m+6$	No
$(3m-3)(2m-2)$	$6m^2-12m+6$	No
$(3m-2)(2m-3)$	$6m^2-13m+6$	Yes

$\therefore\ 6m^2-13m+6=(3m-2)(2m-3)$

Method 2: Breaking Up the Middle Term

$$\begin{aligned}6m^2-13m+6&=6m^2-4m-9m+6\\&=2m(3m-2)-3(3m-2)\\&=(3m-2)(2m-3)\end{aligned}$$

b. Textbook questions 23, 25, 27, 29, 31, 33, 35, 37, and 39 of "Practice," pp. 130 and 131

23. Method 1: Using Guess and Check

Possible Factors	Trinomial	Is the Guess Correct?
$(4t+3)(t+1)$	$4t^2+7t+3$	No
$(4t+1)(t+3)$	$4t^2+13t+3$	No
$(2t+1)(2t+3)$	$4t^2+8t+3$	Yes

$\therefore\ 4t^2+8t+3=(2t+1)(2t+3)$

Method 2: Breaking Up the Middle Term

$$\begin{aligned}4t^2+8t+3&=4t^2+2t+6t+3\\&=2t(2t+1)+3(2t+1)\\&=(2t+1)(2t+3)\end{aligned}$$

25. No two integers have a product of -10 and a sum of 2; therefore, the trinomial $5t^2+2t-2$ cannot be factored.

27. Method 1: Using Guess and Check

$$\begin{aligned}8y^2-22y+12&=2\left(4y^2-11y+6\right)\\&=2(4y-3)(y-2)\end{aligned}$$

Method 2: Breaking Up the Middle Term

$$\begin{aligned}8y^2-22y+12&=2\left(4y^2-11y+6\right)\\&=2\left(4y^2-8y-3y+6\right)\\&=2\left[4y(y-2)-3(y-2)\right]\\&=2(y-2)(4y-3)\end{aligned}$$

29. Method 1: Using Guess and Check

$$\begin{aligned}6r^2+15r+9&=3\left(2r^2+5r+3\right)\\&=3(2r+3)(r+1)\end{aligned}$$

Method 2: Breaking Up the Middle Term

$$\begin{aligned}6r^2+15r+9&=3\left(2r^2+5r+3\right)\\&=3\left(2r^2+2r+3r+3\right)\\&=3\left[2r(r+1)+3(r+1)\right]\\&=3(r+1)(2r+3)\end{aligned}$$

Section 1: Activity 2 (continued)

31. **Method 1: Using Guess and Check**

$$\begin{aligned} 4x^2-18x-10 &= 2\left(2x^2-9x-5\right) \\ &= 2(2x+1)(x-5) \end{aligned}$$

Method 2: Breaking Up the Middle Term

$$\begin{aligned} 4x^2-18x-10 &= 2\left(2x^2-9x-5\right) \\ &= 2\left(2x^2-10x+x-5\right) \\ &= 2\left[2x(x-5)+1(x-5)\right] \\ &= 2(x-5)(2x+1) \end{aligned}$$

33. **Method 1: Using Guess and Check**

$$\begin{aligned} 2t^3+9t^2+4t &= t\left(2t^2+9t+4\right) \\ &= t(2t+1)(t+4) \end{aligned}$$

Method 2: Breaking Up the Middle Term

$$\begin{aligned} 2t^3+9t^2+4t &= t\left(2t^2+9t+4\right) \\ &= t\left(2t^2+8t+t+4\right) \\ &= t\left[2t(t+4)+1(t+4)\right] \\ &= t(t+4)(2t+1) \end{aligned}$$

35. **Method 1: Using Guess and Check**

$$\begin{aligned} 12r^2+27r+15 &= 3\left(4r^2+9r+5\right) \\ &= 3(4r+5)(r+1) \end{aligned}$$

Method 2: Breaking Up the Middle Term

$$\begin{aligned} 12r^2+27r+15 &= 3\left(4r^2+9r+5\right) \\ &= 3\left(4r^2+4r+5r+5\right) \\ &= 3\left[4r(r+1)+5(r+1)\right] \\ &= 3(r+1)(4r+5) \end{aligned}$$

37. **Method 1: Using Guess and Check**

$$6+5y-4y^2 = (2-y)(3+4y)$$

Method 2: Breaking Up the Middle Term

$$\begin{aligned} 6+5y-4y^2 &= 6-3y+8y-4y^2 \\ &= 3(2-y)+4y(2-y) \\ &= (2-y)(3+4y) \end{aligned}$$

39. $12+18t+8t^2 = 2\left(6+9t+4t^2\right)$

Note: $6+9t+4t^2$ cannot be factored.

c. Textbook questions 54 and 55 of "Applications and Problem Solving," p. 131

54. a. $(3x+1)(x+5)=3x^2+16x+5$

$(3x-1)(x-5)=3x^2-16x+5$

$(3x+5)(x+1)=3x^2+8x+5$

$(3x-5)(x-1)=3x^2-8x+5$

Therefore, k can be ± 16 or ± 8.

b. $(5x+1)(x+8)=5x^2+41x+8$

$(5x-1)(x-8)=5x^2-41x+8$

$(5x+2)(x+4)=5x^2+22x+8$

$(5x-2)(x-4)=5x^2-22x+8$

$(5x+4)(x+2)=5x^2+14x+8$

$(5x-4)(x-2)=5x^2-14x+8$

$(5x+8)(x+1)=5x^2+13x+8$

$(5x-8)(x-1)=5x^2-13x+8$

Therefore, k can be ± 41, ± 22, ± 14, or ± 13.

c. $(3x+1)(x-2)=3x^2-5x-2$

$(3x-1)(x+2)=3x^2+5x-2$

$(3x+2)(x-1)=3x^2-x-2$

$(3x-2)(x+1)=3x^2+x-2$

Therefore, k can be ± 5 or ± 1.

d. $(4x+1)(x-9)=4x^2-35x-9$

$(4x-1)(x+9)=4x^2+35x-9$

$(4x+3)(x-3)=4x^2-9x-9$

$(4x-3)(x+3)=4x^2+9x-9$

$(4x+9)(x-1)=4x^2+5x-9$

$(4x-9)(x+1)=4x^2-5x-9$

$(2x+1)(2x-9)=4x^2-16x-9$

$(2x-1)(2x+9)=4x^2+16x-9$

$(2x+3)(2x-3)=4x^2-9$

Therefore, k can be ± 35, ± 9, ± 5, ± 16, or 0.

Section 1: Activity 2 (continued)

55. Answers will vary. Sample answers are given.

a. $(2x+1)(x+1)=2x^2+3x+1$

$(2x-1)(x+2)=2x^2+3x-2$

$(2x-3)(x+3)=2x^2+3x-9$

Therefore, k can be 1, -2, or -9.

b. $(3x-2)(x-2)=3x^2-8x+4$

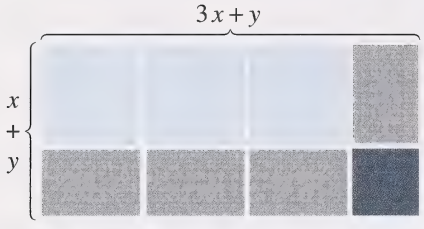
$(3x+1)(x-3)=3x^2-8x-3$

$(3x+4)(x-4)=3x^2-8x-16$

Therefore, k can be 4, -3, or -16.

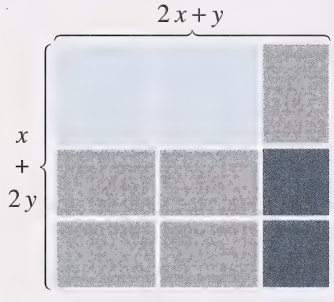
17. Textbook question 52 of "Applications and Problem Solving," p. 131

52. a.



$\therefore\ 3x^2+4xy+y^2=(x+y)(3x+y)$

b.



$\therefore\ 2x^2+5xy+2y^2=(x+2y)(2x+y)$

18. Textbook questions 41, 43, 45, 47, and 49 of "Practice," p. 131

41. Method 1: Using Guess and Check

$6m^2+mn-2n^2=(2m-n)(3m+2n)$

Method 2: Breaking Up the Middle Term

$$\begin{aligned}6m^2+mn-2n^2&=6m^2-3mn+4mn-2n^2\\&=3m(2m-n)+2n(2m-n)\\&=(2m-n)(3m+2n)\end{aligned}$$

43. Method 1: Using Guess and Check

$$10a^2 - 3ab - b^2 = (2a - b)(5a + b)$$

Method 2: Breaking Up the Middle Term

$$\begin{aligned} 10a^2 - 3ab - b^2 &= 10a^2 - 5ab + 2ab - b^2 \\ &= 5a(2a - b) + b(2a - b) \\ &= (2a - b)(5a + b) \end{aligned}$$

45. Method 1: Using Guess and Check

$$6c^2 + 13cd + 2d^2 = (6c + d)(c + 2d)$$

Method 2: Breaking Up the Middle Term

$$\begin{aligned} 6c^2 + 13cd + 2d^2 &= 6c^2 + 12cd + cd + 2d^2 \\ &= 6c(c + 2d) + d(c + 2d) \\ &= (c + 2d)(6c + d) \end{aligned}$$

47. Method 1: Using Guess and Check

$$\begin{aligned} 6x^2 - 9xy + 3y^2 &= 3\left(2x^2 - 3xy + y^2\right) \\ &= 3(2x - y)(x - y) \end{aligned}$$

Method 2: Breaking Up the Middle Term

$$\begin{aligned} 6x^2 - 9xy + 3y^2 &= 3\left(2x^2 - 3xy + y^2\right) \\ &= 3\left(2x^2 - 2xy - xy + y^2\right) \\ &= 3\left[2x(x - y) - y(x - y)\right] \\ &= 3(x - y)(2x - y) \end{aligned}$$

49. Method 1: Using Guess and Check

$$\begin{aligned} 4y^2 + 4xy - 8x^2 &= 4\left(y^2 + xy - 2x^2\right) \\ &= 4(y - x)(y + 2x) \end{aligned}$$

Method 2: Breaking Up the Middle Term

$$\begin{aligned} 4y^2 + 4xy - 8x^2 &= 4\left(y^2 + xy - 2x^2\right) \\ &= 4\left(y^2 - xy + 2xy - 2x^2\right) \\ &= 4\left[y(y - x) + 2x(y - x)\right] \\ &= 4(y - x)(y + 2x) \end{aligned}$$

19. Textbook questions 27, 30, 31, 38, 40, 41, 43, and 44 of "Practice," p. 133

27. $y^2 - 144 = (y + 12)(y - 12)$

30.
$$\begin{aligned} 2x^2 - 32 &= 2\left(x^2 - 16\right) \\ &= 2(x + 4)(x - 4) \end{aligned}$$

31. $y^2 + 36$ cannot be factored.

38.
$$\begin{aligned} y^3 - 36y &= y\left(y^2 - 36\right) \\ &= y(y + 6)(y - 6) \end{aligned}$$

Section 1: Activity 2 (continued)

40. $36x^2+100y^2=4\left(9x^2+25y^2\right)$

41. $3x^3-48x=3x\left(x^2-16\right)$

$=3x(x+4)(x-4)$

43. $81x^2-144=9\left(9x^2-16\right)$

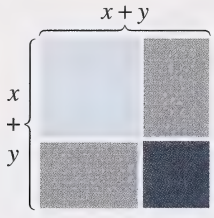
$=9(3x+4)(3x-4)$

44. $3b^2-300=3\left(b^2-100\right)$

$=3(b+10)(b-10)$

20. a. Textbook question 45 of "Applications and Problem Solving," p. 134

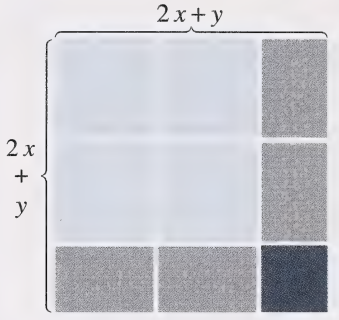
45. a.



$\therefore\ x^2+2xy+y^2=(x+y)(x+y)$

$=(x+y)^2$

b.



$\therefore\ 4x^2+4xy+y^2=(2x+y)(2x+y)$

$=(2x+y)^2$

b. Textbook questions 28, 29, 32, 34, 37, and 39 of "Practice," p. 133

28. $25x^2+5y+1$ cannot be factored.

29. $9a^2-24a+16=(3a-4)^2$

32. $3x^2+6x+3=3\left(x^2+2x+1\right)$

$=3(x+1)^2$

34. $4p^2+20pq+25q^2=(2p+5q)^2$

37. $100x^2+10x+1$ cannot be factored.

39. $y^3-18y^2+81y=y\left(y^2-18y+81\right)$

$=y(y-9)^2$

c. Textbook question 55 of "Applications and Problem Solving," p. 134

55. a. $(x+4)^2 = x^2 + 8x + 16$

$(x-4)^2 = x^2 - 8x + 16$

Therefore, k can be ± 8.

b. $(3x+7)^2 = 9x^2 + 42x + 49$

$(3x-7)^2 = 9x^2 - 42x + 49$

Therefore, k can be ± 42.

c. $(x+2)^2 = x^2 + 4x + 4$

Therefore, $k = 4$.

d. $(2x-3)^2 = 4x^2 - 12x + 9$

Therefore, $k = 9$.

e. $(5x+4)^2 = 25x^2 + 40x + 16$

Therefore, $k = 25$.

f. $(4x-3y)^2 = 16x^2 - 24xy + 9y^2$

Therefore, $k = 16$.

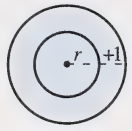
21. Textbook questions 1 to 5 of Investigation 1, "Writing Expressions for Areas," pp. 138 and 139

1. $A = \pi r^2$

2. The inner radius is r.

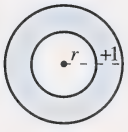
Each ring of seats is about 1 m wide; therefore, the outer radius is $r+1$.

3. a. The following diagram shows the area of the stage plus the first ring of seats:



$A = \pi(r+1)^2$

To find the area of the first ring of seats, subtract the area of the stage from the area of the circle that includes the stage and the first ring of seats.



$\therefore A = \pi(r+1)^2 - \pi r^2$

Section 1: Activity 2 (continued)

b. $A = \pi(r+1)^2 - \pi r^2$

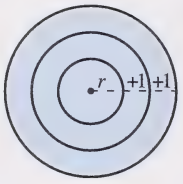
$= \pi\left(r^2 + 2r + 1\right) - \pi r^2$

$= \pi r^2 + 2\pi r + \pi - \pi r^2$

$= 2\pi r + \pi$

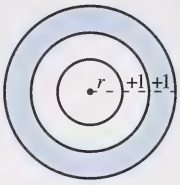
$= \pi(2r+1)$

4. a. The following diagram shows the area of the stage plus the first two rings of seats:



$A = \pi(r+2)^2$

To find the area of the second ring of seats, subtract the area of the stage and the first ring of seats from the area of the circle that includes the stage and the first two rings of seats.



$A = \pi(r+2)^2 - \pi(r+1)^2$

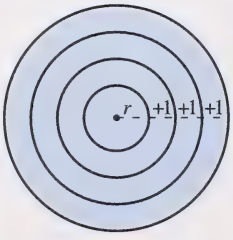
$= \pi\left(r^2 + 4r + 4\right) - \pi\left(r^2 + 2r + 1\right)$

$= \pi r^2 + 4\pi r + 4\pi - \pi r^2 - 2\pi r - \pi$

$= 2\pi r + 3\pi$

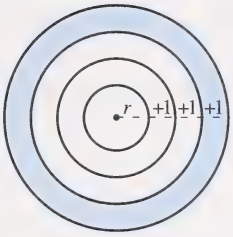
$= \pi(2r+3)$

b. The following diagram shows the area of the stage plus the first three rings of seats:



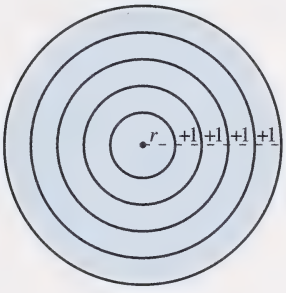
$A = \pi(r+3)^2$

To find the area of the third ring of seats, subtract the area of the stage and the first two rings of seats from the area of the circle that includes the stage and the first three rings of seats.



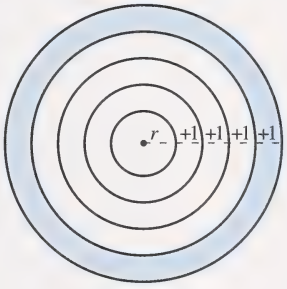
$$\begin{aligned}A &= \pi(r+3)^2 - \pi(r+2)^2 \\ &= \pi\left(r^2+6r+9\right) - \pi\left(r^2+4r+4\right) \\ &= \pi r^2 + 6\pi r + 9\pi - \pi r^2 - 4\pi r - 4\pi \\ &= 2\pi r + 5\pi \\ &= \pi(2r+5)\end{aligned}$$

c. The following diagram shows the area of the stage plus the first four rings of seats:



$$A = \pi(r+4)^2$$

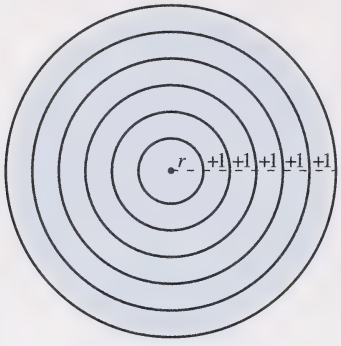
To find the area of the fourth ring of seats, subtract the area of the stage and the first three rings of seats from the area of the circle that includes the stage and the first four rings of seats.



$$\begin{aligned}A &= \pi(r+4)^2 - \pi(r+3)^2 \\ &= \pi\left(r^2+8r+16\right) - \pi\left(r^2+6r+9\right) \\ &= \pi r^2 + 8\pi r + 16\pi - \pi r^2 - 6\pi r - 9\pi \\ &= 2\pi r + 7\pi \\ &= \pi(2r+7)\end{aligned}$$

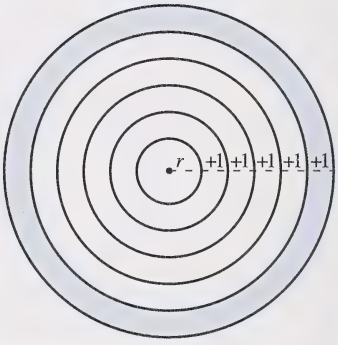
Section 1: Activity 2 (continued)

d. The following diagram shows the area of the stage plus the first five rings of seats:



$$A = \pi(r+5)^2$$

To find the area of the fifth ring of seats, subtract the area of the stage and the first four rings of seats from the area of the circle that includes the stage and the first five rings of seats.



$$\begin{aligned} A &= \pi(r+5)^2 - \pi(r+4)^2 \\ &= \pi\left(r^2+10r+25\right) - \pi\left(r^2+8r+16\right) \\ &= \pi r^2 + 10\pi r + 25\pi - \pi r^2 - 8\pi r - 16\pi \\ &= 2\pi r + 9\pi \\ &= \pi(2r+9) \end{aligned}$$

5. a. area of first ring of seats: $\pi(2r+1)$
area of second ring of seats: $\pi(2r+3)$
area of third ring of seats: $\pi(2r+5)$
area of fourth ring of seats: $\pi(2r+7)$
area of fifth ring of seats: $\pi(2r+9)$

The last term of the binomial factor is two times the position of the seats, minus 1.

b. $A = \pi\left[2r+(2n-1)\right]$

c. $\begin{aligned} A &= \pi\left[2r+(2n-1)\right] \\ &= \pi\left[2r+(2\times 8-1)\right] \\ &= \pi(2r+15) \end{aligned}$

Section 1: Activity 3

1. a.
$$\begin{array}{r} 435 \\ 3\overline{)1306} \\ \underline{12} \\ 10 \\ \underline{9} \\ 16 \\ \underline{15} \\ 1 \end{array}$$

No, 3 is not a factor.

b.
$$\begin{array}{r} 3286 \\ 3\overline{)9858} \\ \underline{9} \\ 08 \\ \underline{6} \\ 25 \\ \underline{24} \\ 18 \\ \underline{18} \\ 0 \end{array}$$

Yes, 3 is a factor.

c.
$$\begin{array}{r} 1947 \\ 3\overline{)5841} \\ \underline{3} \\ 28 \\ \underline{27} \\ 14 \\ \underline{12} \\ 21 \\ \underline{21} \\ 0 \end{array}$$

Yes, 3 is a factor.

2. a. Textbook questions a. and b. of "Explore: Use Algebra Tiles," p. 148

a. The other dimension is $x+5$.

b. The area is $2x(x+5)=2x^2+10x$.

b. Textbook questions 1 and 2 of "Inquire," p. 148

1. $\dfrac{2x^2+10x}{2x}$

2. Each term of the numerator could be divided by the denominator.

$$\begin{aligned}\frac{2x^2+10x}{2x} &= \frac{2x^2}{2x}+\frac{10x}{2x} \\ &= x+5\end{aligned}$$

3. Textbook questions 11 to 17 of "Practice," p. 152

11.
$$\begin{aligned}\frac{2x+18}{2} &= \frac{2x}{2}+\frac{18}{2} \\ &= x+9\end{aligned}$$

12.
$$\begin{aligned}\frac{10a-15b}{5} &= \frac{10a}{5}-\frac{15b}{5} \\ &= 2a-3b\end{aligned}$$

13.
$$\begin{aligned}\frac{6x^2-18x-24}{-6} &= \frac{6x^2}{-6}-\frac{18x}{-6}-\frac{24}{-6} \\ &= -x^2+3x+4\end{aligned}$$

14.
$$\begin{aligned}\frac{14a+7b-7}{7} &= \frac{14a}{7}+\frac{7b}{7}-\frac{7}{7} \\ &= 2a+b-1\end{aligned}$$

Section 1: Activity 3 (continued)

15. $\dfrac{4y^2-16y}{4y}=\dfrac{4y^2}{4y}-\dfrac{16y}{4y}$

$=y-4$

Restriction: $y\neq 0$

16. $\dfrac{3a^2b-12ab^2}{-3ab}=\dfrac{3a^2b}{-3ab}-\dfrac{12ab^2}{-3ab}$

$=-a+4b$

Restrictions: $a\neq 0$ and $b\neq 0$

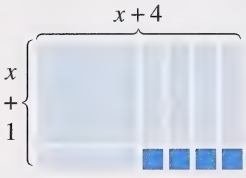
17. $\dfrac{12x^2y^3-15x^4y^2-18x^5y^4}{-3x^2y^2}=\dfrac{12x^2y^3}{-3x^2y^2}-\dfrac{15x^4y^2}{-3x^2y^2}-\dfrac{18x^5y^4}{-3x^2y^2}$

$=-4y+5x^2+6x^3y^2$

Restrictions: $x\neq 0$ and $y\neq 0$

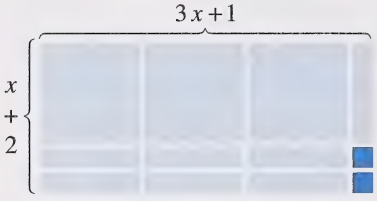
4. Textbook question 67 of "Applications and Problem Solving," p. 153

67. a.



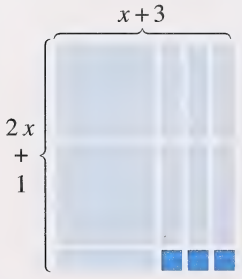
$\therefore \dfrac{x^2+5x+4}{x+1}=x+4$

b.



$\therefore \dfrac{3x^2+7x+2}{x+2}=3x+1$

c.



$\therefore \dfrac{2x^2+7x+3}{2x+1}=x+3$

5. Textbook questions 19, 21, 23, 25, 27, 29, 31, 33, 35, and 37 of "Practice," p. 152

19.
$$\begin{array}{r} a-2 \\ a-5\overline{)a^2-7a+10} \\ \underline{a^2-5a} \\ -2a+10 \\ \underline{-2a+10} \\ 0 \end{array}$$

Restriction: $a \neq 5$

21.
$$\begin{array}{r} t-2 \\ t+2\overline{)t^2+0t-4} \\ \underline{t^2+2t} \\ -2t-4 \\ \underline{-2t-4} \\ 0 \end{array}$$

Restriction: $t \neq -2$

23.
$$\begin{array}{r} t^2-t-1 \\ t+4\overline{)t^3+3t^2-5t-4} \\ \underline{t^3+4t^2} \\ -t^2-5t \\ \underline{-t^2-4t} \\ -t-4 \\ \underline{-t-4} \\ 0 \end{array}$$

Restriction: $t \neq -4$

25.
$$\begin{array}{r} x^2+5x-8 \\ x-1\overline{)x^3+4x^2-13x+8} \\ \underline{x^3-x^2} \\ 5x^2-13x \\ \underline{5x^2-5x} \\ -8x+8 \\ \underline{-8x+8} \\ 0 \end{array}$$

Restriction: $x \neq 1$

27.
$$\begin{array}{r} y^2-2 \\ y-4\overline{)y^3-4y^2-2y+8} \\ \underline{y^3-4y^2} \\ 0y^2-2y+8 \\ \underline{0y^2-2y+8} \\ 0 \end{array}$$

Restriction: $y \neq 4$

29.
$$\begin{array}{r} x-3 \\ x^2+0x+4\overline{)x^3-3x^2+4x-12} \\ \underline{x^3+0x^2+4x} \\ -3x^2+0x-12 \\ \underline{-3x^2+0x-12} \\ 0 \end{array}$$

Restrictions: none

31.
$$\begin{array}{r} a-1 \\ a^2+0a-4\overline{)a^3-a^2-4a+4} \\ \underline{a^3+0a^2-4a} \\ -a^2+0a+4 \\ \underline{-a^2+0a+4} \\ 0 \end{array}$$

Restrictions: $a \neq \pm 2$

33.
$$\begin{array}{r} y+3 \\ 3y-1\overline{)3y^2+8y-3} \\ \underline{3y^2-y} \\ 9y-3 \\ \underline{9y-3} \\ 0 \end{array}$$

Restriction: $y \neq \frac{1}{3}$

Section 1: Activity 3 (continued)

35.
$$\begin{array}{r} 2t-3 \\ 2t+1\overline{)4t^2-4t-3} \\ \underline{4t^2+2t} \\ -6t-3 \\ \underline{-6t-3} \\ 0 \end{array}$$

Restriction: $t \neq -\frac{1}{2}$

37.
$$\begin{array}{r} 2s+3 \\ 5s+3\overline{)10s^2+21s+9} \\ \underline{10s^2+6s} \\ 15s+9 \\ \underline{15s+9} \\ 0 \end{array}$$

Restriction: $s \neq -\frac{3}{5}$

6. Textbook question 68 of "Applications and Problem Solving," p. 153

68. a. $\dfrac{2x^2+18x-72}{2x-6}$

b.
$$\begin{array}{r} x+12 \\ 2x-6\overline{)2x^2+18x-72} \\ \underline{2x^2-6x} \\ 24x-72 \\ \underline{24x-72} \\ 0 \end{array}$$

c. Length $= 2x-6$
$= 2(140)-6$
$= 280-6$
$= 274$

Width $= x+12$
$= 140+12$
$= 152$

The dimensions of the table are 274 cm by 152 cm.

7. Textbook questions 47, 49, 51, 53, and 55 of "Practice," p. 152

47.
$$\begin{array}{r} t \\ t+4\overline{)t^2+4t+2} \\ \underline{t^2+4t} \\ 0t+2 \end{array}$$

Restriction: $t \neq -4$

$\therefore \dfrac{P}{D} = Q+\dfrac{R}{D}$

$\dfrac{t^2+4t+2}{t+4} = t+\dfrac{2}{t+4}$

$\therefore P = DQ+R$

$t^2+4t+2 = (t+4)t+2$

49.
$$\begin{array}{r} y-3 \\ y-4\overline{)\,y^2-7y+10} \\ \underline{y^2-4y} \\ -3y+10 \\ \underline{-3y+12} \\ -2 \end{array}$$

Restriction: $y \neq 4$

$$\therefore \frac{P}{D} = Q + \frac{R}{D} \qquad \therefore P = DQ + R$$

$$\frac{y^2-7y+10}{y-4} = y-3-\frac{2}{y-4} \qquad y^2-7y+10 = (y-4)(y-3)-2$$

51.
$$\begin{array}{r} 4a+1 \\ a-2\overline{)\,4a^2-7a-7} \\ \underline{4a^2-8a} \\ a-7 \\ \underline{a-2} \\ -5 \end{array}$$

Restriction: $a \neq 2$

$$\therefore \frac{P}{D} = Q + \frac{R}{D} \qquad \therefore P = DQ + R$$

$$\frac{4a^2-7a-7}{a-2} = 4a+1-\frac{5}{a-2} \qquad 4a^2-7a-7 = (a-2)(4a+1)-5$$

53.
$$\begin{array}{r} c+2 \\ 5c+4\overline{)\,5c^2+14c+11} \\ \underline{5c^2+4c} \\ 10c+11 \\ \underline{10c+8} \\ 3 \end{array}$$

Restriction: $c \neq -\frac{4}{5}$

$$\therefore \frac{P}{D} = Q + \frac{R}{D} \qquad \therefore P = DQ + R$$

$$\frac{5c^2+14c+11}{5c+4} = c+2+\frac{3}{5c+4} \qquad 5c^2+14c+11 = (5c+4)(c+2)+3$$

Section 1: Activity 3 (continued)

55.
$$\begin{array}{r} 4n-3 \\ 2n-3\overline{)\,8n^2-18n+13} \\ \underline{8n^2-12n} \\ -6n+13 \\ \underline{-6n+9} \\ 4 \end{array}$$

Restriction: $n \neq \frac{3}{2}$

$$\therefore \frac{P}{D} = Q + \frac{R}{D} \qquad \therefore P = DQ + R$$

$$\frac{8n^2-18n+13}{2n-3} = 4n-3+\frac{4}{2n-3} \qquad 8n^2-18n+13 = (2n-3)(4n-3)+4$$

8. Textbook questions 57, 59, 61, and 63 of "Practice," p. 152

57.
$$\begin{array}{r} 2x^2+3 \\ 3x+2\overline{)\,6x^3+4x^2+9x+8} \\ \underline{6x^3+4x^2} \\ 0x^2+9x+8 \\ \underline{0x^2+9x+6} \\ 2 \end{array}$$

Restriction: $x \neq -\frac{2}{3}$

$$\therefore \frac{P(x)}{D(x)} = Q(x) + \frac{R}{D(x)} \qquad \therefore P(x) = D(x)Q(x) + R$$

$$\frac{6x^3+4x^2+9x+8}{3x+2} = 2x^2+3+\frac{2}{3x+2} \qquad 6x^3+4x^2+9x+8 = (3x+2)\left(2x^2+3\right)+2$$

59.
$$\begin{array}{r} 2f^2-3 \\ 6f-5\overline{)\,12f^3-10f^2-18f+25} \\ \underline{12f^3-10f^2} \\ 0f^2-18f+25 \\ \underline{0f^2-18f+15} \\ 10 \end{array}$$

Restriction: $f \neq \frac{5}{6}$

$$\therefore \frac{P(x)}{D(x)} = Q(x) + \frac{R}{D(x)}$$

$$\frac{12f^3 - 10f^2 - 18f + 25}{6f - 5} = 2f^2 - 3 + \frac{10}{6f - 5}$$

$$\therefore P(x) = D(x)Q(x) + R$$

$$12f^3 - 10f^2 - 18f + 25 = (6f - 5)\left(2f^2 - 3\right) + 10$$

61.
$$\begin{array}{r} 3n + 7 \\ 2n^2 + 0n + 3 \overline{)\,6n^3 + 14n^2 + 9n + 12} \\ \underline{6n^3 + 0n^2 + 9n} \\ 14n^2 + 0n + 12 \\ \underline{14n^2 + 0n + 21} \\ -9 \end{array}$$

Restrictions: none

$$\therefore \frac{P(x)}{D(x)} = Q(x) + \frac{R}{D(x)}$$

$$\frac{6n^3 + 14n^2 + 9n + 12}{2n^2 + 3} = 3n + 7 - \frac{9}{2n^2 + 3}$$

$$\therefore P(x) = D(x)Q(x) + R$$

$$6n^3 + 14n^2 + 9n + 12 = \left(2n^2 + 3\right)(3n + 7) - 9$$

63.
$$\begin{array}{r} 2m - 1 \\ 4m^2 + 0m + 1 \overline{)\,8m^3 - 4m^2 + 2m + 0} \\ \underline{8m^3 + 0m^2 + 2m} \\ -4m^2 + 0m + 0 \\ \underline{-4m^2 + 0m - 1} \\ 1 \end{array}$$

Restrictions: none

$$\therefore \frac{P(x)}{D(x)} = Q(x) + \frac{R}{D(x)}$$

$$\frac{8m^3 - 4m^2 + 2m}{4m^2 + 1} = 2m - 1 + \frac{1}{4m^2 + 1}$$

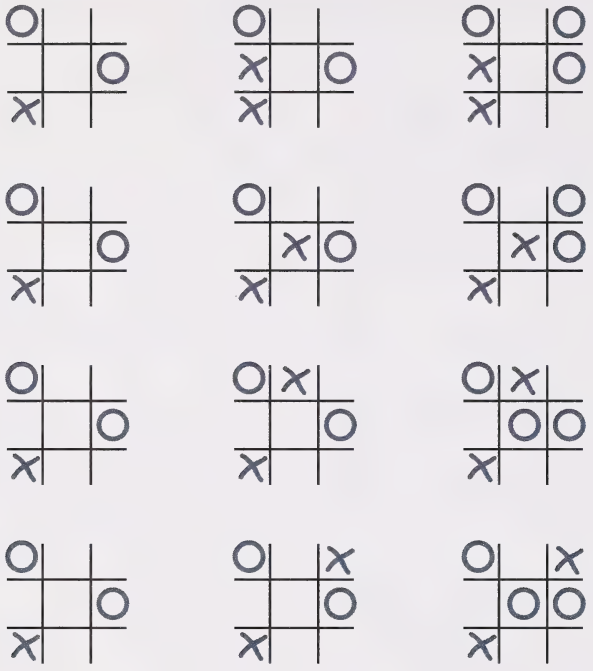
$$\therefore P(x) = D(x)Q(x) + R$$

$$8m^3 - 4m^2 + 2m = \left(4m^2 + 1\right)(2m - 1) + 1$$

Section 1: Activity 3 (continued)

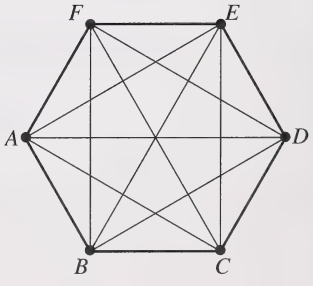
9. Textbook question "LOGIC POWER," p. 109

The four moves that **X** should not play are depicted in the middle column. Should any of these moves be made, **O** can make the move depicted in the last column to ensure victory.



10. Textbook question 5 of "Applications and Problem Solving," p. 115

5. A six-team league could be represented by a regular hexagon in which each team is a vertex. The diagonals and edges connecting the vertices represent matches played between teams.



There are 15 connecting segments in the diagram (9 diagonals, 6 edges). Since each team plays each other 4 times, there are $15 \times 4 = 60$ games in total.

Section 1: Follow-up Activities

Extra Help

1. Textbook questions 21 and 22 of "Review," p. 140

21.

$$\begin{array}{r} x^2-3x-2 \\ \times \quad x-3 \\ \hline -3x^2+9x+6 \\ x^3-3x^2-2x \\ \hline x^3-6x^2+7x+6 \end{array}$$

22.

$$\begin{array}{r} 3t^2-t-1 \\ \times \quad 2t+1 \\ \hline 3t^2-t-1 \\ 6t^3-2t^2-2t \\ \hline 6t^3+t^2-3t-1 \end{array}$$

2. Textbook questions 1 to 4 of "Chapter Check," p. 202

1. $\dfrac{9y^3+6y^2-12y}{3y} = \dfrac{9y^3}{3y}+\dfrac{6y^2}{3y}-\dfrac{12y}{3y}$

$=3y^2+2y-4$

Restriction: $y \neq 0$

2.

$$\begin{array}{r} 4t-3 \\ 3t+1\overline{)12t^2-5t-3} \\ \underline{12t^2+4t} \\ -9t-3 \\ \underline{-9t-3} \\ 0 \end{array}$$

Restriction: $t \neq -\dfrac{1}{3}$

3.

$$\begin{array}{r} x^2+3x+1 \\ x-2\overline{)x^3+x^2-5x+2} \\ \underline{x^3-2x^2} \\ 3x^2-5x \\ \underline{3x^2-6x} \\ x+2 \\ \underline{x-2} \\ 4 \end{array}$$

Restriction: $x \neq 2$

4.

$$\begin{array}{r} n-1 \\ 2n^2+0n+1\overline{)2n^3-2n^2+n+0} \\ \underline{2n^3+0n^2+n} \\ -2n^2+0n+0 \\ \underline{-2n^2-0n-1} \\ 1 \end{array}$$

Restrictions: none

Enrichment

1. Textbook questions 1, 4, 6, 8, and 9 of Investigation 1, "Dividing Polynomials by $x-b$," p. 154

1.
$$\begin{array}{r|rrr} 1 & 1 & 4 & -5 \\ & & 1 & 5 \\ \hline & 1 & 5 & 0 \end{array}$$

$$\therefore \frac{x^2+4x-5}{x-1} = x+5$$

4.
$$\begin{array}{r|rrr} 3 & 2 & -2 & -12 \\ & & 6 & 12 \\ \hline & 2 & 4 & 0 \end{array}$$

$$\therefore \frac{2t^2-2t-12}{t-3} = 2t+4$$

6.
$$\begin{array}{r|rrrr} 2 & 4 & -10 & 3 & 2 \\ & & 8 & -4 & -2 \\ \hline & 4 & -2 & -1 & 0 \end{array}$$

$$\therefore \frac{4r^3-10r^2+3r+2}{r-2} = 4r^2-2r-1$$

8.
$$\begin{array}{r|rrrr} 5 & 3 & -16 & 0 & 25 \\ & & 15 & -5 & -25 \\ \hline & 3 & -1 & -5 & 0 \end{array}$$

$$\therefore \frac{3c^3-16c^2+25}{c-5} = 3c^2-c-5$$

9.
$$\begin{array}{r|rrrrr} 1 & 1 & -1 & 0 & 1 & -1 \\ & & 1 & 0 & 0 & 1 \\ \hline & 1 & 0 & 0 & 1 & 0 \end{array}$$

$$\therefore \frac{x^4-x^3+x-1}{x-1} = x^3+1$$

2. Textbook questions 2, 3, 5, and 7 of Investigation 1, "Dividing Polynomials by $x-b$," p. 154

2.
$$\begin{array}{r|rrr} -2 & 1 & 8 & 12 \\ & & -2 & -12 \\ \hline & 1 & 6 & 0 \end{array}$$

$$\therefore \frac{y^2+8y+12}{y+2} = y+6$$

3.
$$\begin{array}{r|rrr} -3 & 1 & 0 & -9 \\ & & -3 & 9 \\ \hline & 1 & -3 & 0 \end{array}$$

$$\therefore \frac{n^2-9}{n+3} = n-3$$

5.
$$\begin{array}{r|rrr} -2 & 5 & 7 & -6 \\ & & -10 & 6 \\ \hline & 5 & -3 & 0 \end{array}$$

$$\therefore \frac{5n^2+7n-6}{n+2} = 5n-3$$

7.
$$\begin{array}{r|rrrr} -4 & 6 & 24 & 1 & 4 \\ & & -24 & 0 & -4 \\ \hline & 6 & 0 & 1 & 0 \end{array}$$

$$\therefore \frac{6a^3+24a^2+a+4}{a+4} = 6a^2+1$$

3. Textbook questions 11, 13, 15, and 17 of Investigation 1, "Dividing Polynomials by $x-b$," p. 154

11.
$$\begin{array}{r|rrr} -3 & 3 & 16 & 18 \\ & & -9 & -21 \\ \hline & 3 & 7 & -3 \end{array}$$

$$\therefore \frac{3x^2+16x+18}{x+3} = 3x+7+\frac{-3}{x+3}$$
$$= 3x+7-\frac{3}{x+3}$$

13.
$$\begin{array}{r|rrrr} -10 & 2 & 19 & -11 & -15 \\ & & -20 & 10 & 10 \\ \hline & 2 & -1 & -1 & -5 \end{array}$$

$$\therefore \frac{2z^3+19z^2-11z-15}{z+10} = 2z^2-z-1+\frac{-5}{z+10}$$
$$= 2z^2-z-1-\frac{5}{z+10}$$

15.
$$\begin{array}{r|rrrrr} 4 & 1 & -2 & -8 & 2 & -1 \\ & & 4 & 8 & 0 & 8 \\ \hline & 1 & 2 & 0 & 2 & 7 \end{array}$$

$$\therefore \frac{q^4-2q^3-8q^2+2q-1}{q-4} = q^3+2q^2+2+\frac{7}{q-4}$$

17.
$$\begin{array}{r|rrrrr} -2 & 9 & 18 & -1 & 0 & 7 \\ & & -18 & 0 & 2 & -4 \\ \hline & 9 & 0 & -1 & 2 & 3 \end{array}$$

$$\therefore \frac{9y^4+18y^3-y^2+7}{y+2} = 9y^3-y+2+\frac{3}{y+2}$$

Section 2: Activity 1

1. a. $x+2\neq 0$

$x\neq -2$

b. $y^2-1\neq 0$

$y^2\neq 1$

$y\neq 1,\ -1$

c. $a-b\neq 0$

$a\neq b$

2. a. Textbook questions 9, 11, 13, 15, and 17 of "Practice," p. 158

9. $\dfrac{2x}{2x+8}=\dfrac{\overset{1}{\cancel{2}}(x)}{\underset{1}{\cancel{2}}(x+4)}$

$=\dfrac{x}{x+4}$

Restriction: $x\neq -4$

11. $\dfrac{10x}{5x^2-15x}=\dfrac{\overset{1}{\cancel{5x}}(2)}{\underset{1}{\cancel{5x}}(x-3)}$

$=\dfrac{2}{x-3}$

Restrictions: $x\neq 0,\ 3$

13. $\dfrac{6a^2+9a}{12a^2}=\dfrac{\overset{1}{\cancel{3a}}(2a+3)}{\underset{1}{\cancel{3a}}(4a)}$

$=\dfrac{2a+3}{4a}$

Restriction: $a\neq 0$

15. $\dfrac{3t^3+6t^2-15t}{3t}=\dfrac{\overset{1}{\cancel{3t}}\left(t^2+2t-5\right)}{\underset{1}{\cancel{3t}}}$

$=t^2+2t-5$

Restriction: $t\neq 0$

17. $\dfrac{14n^4-4n^3+6n^2+8n}{2n^2}=\dfrac{\overset{1}{\cancel{2n}}\left(7n^3-2n^2+3n+4\right)}{\underset{1}{\cancel{2n}}(n)}$

$=\dfrac{7n^3-2n^2+3n+4}{n}$

Restriction: $n\neq 0$

b. Textbook questions 19, 21, 23, 25, and 27 of "Practice," p. 158

19. $\dfrac{4x+4y}{5x+5y} = \dfrac{4\cancel{(x+y)}}{5\cancel{(x+y)}}$

$= \dfrac{4}{5}$

Restriction: $x \neq -y$

21. $\dfrac{6t-36}{t-6} = \dfrac{6\cancel{(t-6)}}{\cancel{(t-6)}}$

$= 6$

Restriction: $t \neq 6$

23. $\dfrac{5x-10}{3x-6} = \dfrac{5\cancel{(x-2)}}{3\cancel{(x-2)}}$

$= \dfrac{5}{3}$

Restriction: $x \neq 2$

25. $\dfrac{2x^2-2x}{2x^2+2x} = \dfrac{\cancel{2x}(x-1)}{\cancel{2x}(x+1)}$

$= \dfrac{x-1}{x+1}$

Restrictions: $x \neq -1,\ 0$

27. $\dfrac{5xy+10x}{2y^2+4y} = \dfrac{5x\cancel{(y+2)}}{2y\cancel{(y+2)}}$

$= \dfrac{5x}{2y}$

Restrictions: $y \neq -2,\ 0$

c. Textbook questions 29, 31, 33, 35, and 37 of "Practice," p. 158

29. $\dfrac{y^2+10y+25}{y+5} = \dfrac{(y+5)^2}{y+5}$

$= \dfrac{\cancel{(y+5)}(y+5)}{\cancel{y+5}}$

$= y+5$

Restriction: $y \neq -5$

31. $\dfrac{r^2-4}{5r+10} = \dfrac{(r-2)\cancel{(r+2)}}{5\cancel{(r+2)}}$

$= \dfrac{r-2}{5}$

Restriction: $r \neq -2$

Section 2: Activity 1 (continued)

33. $\dfrac{x^2-9}{2x^2y-6xy} = \dfrac{(x+3)\cancel{(x-3)}^{1}}{2xy\cancel{(x-3)}_{1}}$

$= \dfrac{x+3}{2xy}$

Restrictions: $x \neq 0,\ 3$ and $y \neq 0$

35. $\dfrac{3t^2-8t+4}{6t^2-4t} = \dfrac{(t-2)\cancel{(3t-2)}^{1}}{2t\cancel{(3t-2)}_{1}}$

$= \dfrac{t-2}{2t}$

Restrictions: $t \neq 0,\ \dfrac{2}{3}$

37. $\dfrac{5x^2+3xy-2y^2}{3x^2+3xy} = \dfrac{(5x-2y)\cancel{(x+y)}^{1}}{3x\cancel{(x+y)}_{1}}$

$= \dfrac{5x-2y}{3x}$

Restrictions: $x \neq 0,\ -y$

d. Textbook questions 39, 41, 43, 45, and 47 of "Practice," p. 158

39. $\dfrac{a^2-a-12}{a^2-9a+20} = \dfrac{\cancel{(a-4)}^{1}(a+3)}{\cancel{(a-4)}_{1}(a-5)}$

$= \dfrac{a+3}{a-5}$

Restrictions: $a \neq 4,\ 5$

41. $\dfrac{y^2-8y+15}{y^2-25} = \dfrac{(y-3)\cancel{(y-5)}^{1}}{(y+5)\cancel{(y-5)}_{1}}$

$= \dfrac{y-3}{y+5}$

Restrictions: $y \neq \pm 5$

43. $\dfrac{n^2-n-2}{n^2+n-6} = \dfrac{\cancel{(n-2)}^{1}(n+1)}{\cancel{(n-2)}_{1}(n+3)}$

$= \dfrac{n+1}{n+3}$

Restrictions: $n \neq -3,\ 2$

45. $\dfrac{2t^2-t-1}{t^2-3t+2} = \dfrac{(2t+1)\cancel{(t-1)}^{1}}{(t-2)\cancel{(t-1)}_{1}}$

$= \dfrac{2t+1}{t-2}$

Restrictions: $t \neq 1,\ 2$

47. $\dfrac{6x^2-13x+6}{8x^2-6x-9}=\dfrac{(3x-2)\cancel{(2x-3)}}{(4x+3)\cancel{(2x-3)}}$

$=\dfrac{3x-2}{4x+3}$

Restrictions: $x\neq -\dfrac{3}{4},\ \dfrac{3}{2}$

3. a. $\dfrac{8a}{-3}=\dfrac{-8a}{3}$ or $-\dfrac{8a}{3}$

b. $\dfrac{5x^2y}{-6x}=\dfrac{\cancel{x}(5xy)}{-6\cancel{x}}$

$=\dfrac{5xy}{-6}$

$=\dfrac{-5xy}{6}$ or $-\dfrac{5xy}{6}$ **Restriction:** $x\neq 0$

c. $\dfrac{x-1}{-5}=\dfrac{-x+1}{5}$

$=\dfrac{1-x}{5}$

d. $\dfrac{a^2-a}{-3a}=\dfrac{\cancel{a}(a-1)}{-3\cancel{a}}$

$=\dfrac{a-1}{-3}$

$=\dfrac{-a+1}{3}$

$=\dfrac{1-a}{3}$ **Restriction:** $a\neq 0$

4. a. $\dfrac{2x^2+10x}{20+4x}=\dfrac{2x^2+10x}{4x+20}$

$=\dfrac{\cancel{2}x\cancel{(x+5)}}{\cancel{4}\cancel{(x+5)}}$

$=\dfrac{x}{2}$

Restriction: $x\neq -5$

b. $\dfrac{3x-12y}{20y-5x}=\dfrac{3x-12y}{-5x+20y}$

$=\dfrac{3\cancel{(x-4y)}}{-5\cancel{(x-4y)}}$

$=\dfrac{3}{-5}$

$=-\dfrac{3}{5}$

Restriction: $x\neq 4y$

Section 2: Activity 1 (continued)

c.
$$\frac{25-a^2}{2a^2-9a-5}=\frac{-a^2+25}{2a^2-9a-5}$$
$$=\frac{-1\left(a^2-25\right)}{(2a+1)(a-5)}$$
$$=\frac{-1(a+5)\cancel{(a-5)}}{(2a+1)\cancel{(a-5)}}$$
$$=\frac{-a-5}{2a+1}$$

Restrictions: $a \neq -\frac{1}{2}, 5$

d.
$$\frac{m-5}{10-2m}=\frac{m-5}{-2m+10}$$
$$=\frac{\cancel{(m-5)}}{-2\cancel{(m-5)}}$$
$$=\frac{1}{-2}$$
$$=-\frac{1}{2}$$

Restriction: $m \neq 5$

5. a. Textbook question "Explore: Complete the Table," p. 156

	x-value					
Rational Expression	–1	0	1	2	3	4
$\frac{2x+4}{x^2+5x+6}$	1	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{2}{5}$	$\frac{1}{3}$	$\frac{2}{7}$
$\frac{2}{x+3}$	1	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{2}{5}$	$\frac{1}{3}$	$\frac{2}{7}$

b. Textbook questions 1 to 3 of "Inquire," p. 156

1. The two rational expressions are equivalent.

2.
$$\frac{2x+4}{x^2+5x+6}=\frac{2(x+2)}{(x+3)(x+2)}$$

3. The numerator and denominator have a common factor of $x+2$. When the common factors are cancelled, the expression is equal to $\frac{2}{x+3}$.

6. Textbook questions 1 and 2 of Investigation 1, "Shutter Speed," p. 181

1. $\frac{1}{500}\text{ s} < \frac{1}{2}\text{ s}$

Thus, 500 gives a faster shutter speed.

2. a. A high number indicating a fast shutter speed should be used to shoot a "stop action" sports shot.
b. A low number indicating a slow shutter speed can be used to shoot a still life or stationary object.

Section 2: Activity 2

1. a. Textbook questions 23, 25, and 27 of "Practice," p. 163

23. $\frac{3}{x-4} \times \frac{x-4}{6} = \frac{\overset{1}{\cancel{3}} \times \overset{1}{\cancel{(x-4)}}}{\underset{1}{\cancel{(x-4)}} \times \underset{1}{\cancel{(3)}}(2)}$

$= \frac{1}{2}$

Restriction: $x \neq 4$

25. $\frac{5(y-2)}{y+1} \times \frac{y+1}{10} = \frac{\overset{1}{\cancel{5}}(y-2) \times \overset{1}{\cancel{(y+1)}}}{\underset{1}{\cancel{(y+1)}} \times \underset{1}{\cancel{(5)}}(2)}$

$= \frac{y-2}{2}$

Restriction: $y \neq -1$

27. $\frac{4a^2b}{3(a+b)} \div \frac{-8ab^2}{a+b} = \frac{4a^2b}{3(a+b)} \times \frac{a+b}{-8ab^2}$

$= \frac{4a^2b \times (a+b)}{3(a+b) \times \left(-8ab^2\right)}$

$= \frac{\overset{1}{\cancel{4}}(a)\overset{1}{\cancel{(a)}}\overset{1}{\cancel{(b)}} \times \overset{1}{\cancel{(a+b)}}}{\underset{1}{\cancel{3(a+b)}} \times (-2)\underset{1}{\cancel{(4)}}\underset{1}{\cancel{(a)}}\underset{1}{\cancel{(b)}}(b)}$

$= \frac{a}{-6b}$

$= -\frac{a}{6b}$

Restrictions: $a \neq -b$, 0 and $b \neq 0$

Section 2: Activity 2 (continued)

b. Textbook questions 29, 31, 33, and 35 of "Practice," p. 163

29. $\dfrac{4x+4}{3x-3}\times\dfrac{6x-6}{5x+5}=\dfrac{4(x+1)}{3(x-1)}\times\dfrac{6(x-1)}{5(x+1)}$

$$=\frac{4\cancel{(x+1)}\times\cancel{(3)}(2)\cancel{(x-1)}}{\cancel{3(x-1)}\times 5\cancel{(x+1)}}$$

$$=\frac{8}{5}$$

Restrictions: $x\neq\pm 1$

31. $\dfrac{3a+6}{9a^2}\div\dfrac{a+2}{-3a}=\dfrac{3a+6}{9a^2}\times\dfrac{-3a}{a+2}$

$$=\frac{\cancel{3(a+2)}\times(-1)\cancel{(3)(a)}}{\cancel{(3)(3)(a)}(a)\times\cancel{(a+2)}}$$

$$=-\frac{1}{a}$$

Restrictions: $a\neq -2,\ 0$

33. $\dfrac{7y^2}{y^2-9}\times\dfrac{4y+12}{14y^3}=\dfrac{\cancel{7}\cancel{(y^2)}\times(2)\cancel{(2)}\cancel{(y+3)}}{(y-3)\cancel{(y+3)}\times\cancel{(7)}\cancel{(2)}\cancel{(y^2)}\times y}$

$$=\frac{2}{y(y-3)}$$

Restrictions: $y\neq 0,\ \pm 3$

35. $\dfrac{4x-6}{8x^2y}\times\dfrac{4xy}{6x-9}=\dfrac{\overset{1}{\cancel{2}}\overset{1}{\cancel{(2x-3)}}\times\overset{1}{\cancel{(4)}}\overset{1}{\cancel{(x)}}\overset{1}{\cancel{(y)}}}{\underset{1}{\cancel{(2)}}\underset{1}{\cancel{(4)}}\underset{1}{\cancel{(x)}}\underset{1}{\cancel{(x)}}\underset{1}{\cancel{(y)}}\times 3\underset{1}{\cancel{(2x-3)}}}$

$=\dfrac{1}{3x}$

Restrictions: $x\neq 0,\ \dfrac{3}{2}$ and $y\neq 0$

c. Textbook questions 37, 39, and 41 of "Practice," p. 163

37. $\dfrac{x^2+5x+6}{x^2-6x+5}\times\dfrac{x^2+x-30}{x^2+9x+18}=\dfrac{(x+2)\overset{1}{\cancel{(x+3)}}\times\overset{1}{\cancel{(x+6)}}\overset{1}{\cancel{(x-5)}}}{\underset{1}{\cancel{(x-5)}}(x-1)\times\underset{1}{\cancel{(x+6)}}\underset{1}{\cancel{(x+3)}}}$

$=\dfrac{x+2}{x-1}$

Restrictions: $x\neq -6,\ -3,\ 1,\ 5$

39. $\dfrac{m^2-3m-4}{m^2+5m}\div\dfrac{m^2-7m+12}{m^2+2m-15}=\dfrac{m^2-3m-4}{m^2+5m}\times\dfrac{m^2+2m-15}{m^2-7m+12}$

$=\dfrac{\overset{1}{\cancel{(m-4)}}(m+1)\times\overset{1}{\cancel{(m+5)}}\overset{1}{\cancel{(m-3)}}}{m\underset{1}{\cancel{(m+5)}}\times\underset{1}{\cancel{(m-4)}}\underset{1}{\cancel{(m-3)}}}$

$=\dfrac{m+1}{m}$

Restrictions: $m\neq -5,\ 0,\ 3,\ 4$

41. $\dfrac{x^2+3xy}{x^2-xy-42y^2}\times\dfrac{x^2-10xy+21y^2}{x^2-9y^2}=\dfrac{\overset{1}{\cancel{x(x+3y)}}\times\overset{1}{\cancel{(x-7y)}}\overset{1}{\cancel{(x-3y)}}}{\underset{1}{\cancel{(x-7y)}}(x+6y)\times\underset{1}{\cancel{(x+3y)}}\underset{1}{\cancel{(x-3y)}}}$

$=\dfrac{x}{x+6y}$

Restrictions: $x\neq -6y,\ \pm 3y,\ 7y$

Section 2: Activity 2 (continued)

d. Textbook questions 43, 45, and 47 of "Practice," p. 163

43. $\dfrac{12a^2-19a+5}{4a^2-9}\times\dfrac{2a-3}{3a-1}=\dfrac{\overset{1}{\cancel{(3a-1)}}(4a-5)\times\overset{1}{\cancel{(2a-3)}}}{\underset{1}{\cancel{(2a-3)}}(2a+3)\times\underset{1}{\cancel{(3a-1)}}}$

$$=\frac{4a-5}{2a+3}$$

Restrictions: $a\neq\frac{1}{3},\ \pm\frac{3}{2}$

45. $\dfrac{12w^2-5w-2}{8w^2+2w-21}\div\dfrac{12w^2+w-6}{8w^2-2w-15}=\dfrac{12w^2-5w-2}{8w^2+2w-21}\times\dfrac{8w^2-2w-15}{12w^2+w-6}$

$$=\frac{(4w+1)\overset{1}{\cancel{(3w-2)}}\times\overset{1}{\cancel{(2w-3)}}(4w+5)}{\underset{1}{\cancel{(2w-3)}}(4w+7)\times(4w+3)\underset{1}{\cancel{(3w-2)}}}$$

$$=\frac{(4w+1)(4w+5)}{(4w+7)(4w+3)}$$

Restrictions: $w\neq-\frac{7}{4},\ -\frac{5}{4},\ -\frac{3}{4},\ \frac{2}{3},\ \frac{3}{2}$

47. $\dfrac{3x^3+14x^2-5x}{4x^2+7x+3}\div\dfrac{6x^2-2x}{8x^2+2x-3}=\dfrac{3x^3+14x^2-5x}{4x^2+7x+3}\times\dfrac{8x^2+2x-3}{6x^2-2x}$

$$=\frac{\overset{1}{\cancel{x}}\overset{1}{\cancel{(3x-1)}}(x+5)\times(2x-1)\overset{1}{\cancel{(4x+3)}}}{\underset{1}{\cancel{(4x+3)}}(x+1)\times2\underset{1}{\cancel{(x)}}\underset{1}{\cancel{(3x-1)}}}$$

$$=\frac{(x+5)(2x-1)}{2(x+1)}$$

Restrictions: $x\neq-1,\ -\frac{3}{4},\ 0,\ \frac{1}{3},\ \frac{1}{2}$

2. Textbook question 55 of "Applications and Problem Solving," p. 164

55. a. $A_g = \frac{10x}{9} \times \frac{x}{3}$

$= \frac{10x^2}{27}$

b. $A_p = \frac{22x}{9} \times x$

$= \frac{22x^2}{9}$

c. $A_p \div A_g = \frac{22x^2}{9} \div \frac{10x^2}{27}$

$= \frac{22x^2}{9} \times \frac{27}{10x^2}$

$= \frac{\overset{1}{\cancel{2}}(11)\overset{1}{\cancel{x^2}} \times (3)\overset{1}{\cancel{9}}}{\underset{1}{\cancel{9}} \times \underset{1}{\cancel{2}}(5)\underset{1}{\cancel{x^2}}}$

$= \frac{33}{5}$

The penalty area is $\frac{33}{5}$ times greater than the goal box.

d. No. The answer in 55.c. is independent of x.

3. a. Textbook questions 1 and 2 of Investigation 1, "Writing Directions," p. 197

1. To drive from Barry's Bay to Latchford Bridge, passing through Pembroke and not travelling on the same road twice, do the following:

- Drive N on the local road to Bonnechere.
- Then drive NE on Highway 62 to Pembroke.
- Then drive S on Highway 41 to Eganville.
- Then drive SW on Secondary Highway 512 to Foymount.
- Then drive S on Secondary Highway 515 to Latchford Bridge.

Section 2: Activity 2 (continued)

2. To drive from Whitney to Combermere, do the following:

- Drive E on Highway 60, through Madawaska, to Barry's Bay.
- Then drive S on Highway 62 to Combermere.

To drive from Combermere to McArthur Mills, do the following:

- Drive SE on Secondary Highway 515 to Latchford Bridge.
- Then drive S on Secondary Highway 514 to Hardwood Lake.
- Then drive SW on Highway 28 to McArthur Mills.

To drive from McArthur Mills to Dacre, do the following:

- Drive NE on Highway 28 to Hardwood Lake.
- Then drive SE on Highway 28 to Denbigh.
- Then drive NE on Highway 41 to Dacre.

b. Textbook questions 1 and 2 of Investigation 2, "Comparing Routes," p. 197

1. a. The shortest driving distance is the Cobden–Eganville–Foymount–Latchford Bridge–Combermere–Maynooth route. This route is 130 km.

b. The shortest driving distance using at least one primary highway, at least one secondary highway, and no local roads is the Cobden–Pembroke–Killaloe–Foymount–Latchford Bridge–Combermere–Maynooth route. This route is 183 km.

c. The shortest driving distance using only primary highways is the Cobden–Pembroke–Killaloe–Barry's Bay–Combermere–Maynooth route. This route is 158 km.

2. a. The driving time on the Cobden–Pembroke–Killaloe–Barry's Bay–Combermere–Maynooth route is about 1.975 h, which is about 1 h 59 min.

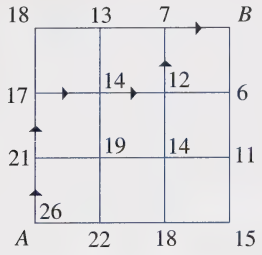
$$\frac{158}{80} = 1.975$$

b. The driving time on the Cobden–Pembroke–Killaloe–Foymount–Latchford Bridge–Combermere–Maynooth route is about 2.6 h, which is about 2 h 36 min.

$$\begin{aligned}\frac{84}{80} + \frac{66}{60} + \frac{33}{80} &= \frac{117}{80} + \frac{66}{60} \\ &\doteq 1.5 + 1.1 \\ &\doteq 2.6\end{aligned}$$

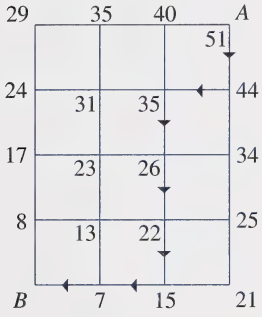
4. Textbook question 1 of "Applications and Problem Solving," p. 123

1. a.



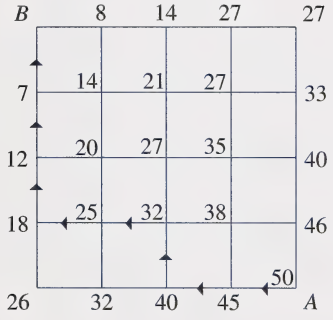
The shortest route takes 26 min. From *A,* proceed north, north, east, east, north, and east.

b.



The shortest route takes 51 min. From *A,* proceed south, west, south, south, south, west, and west.

c.



The shortest route takes 50 min. From *A,* proceed west, west, north, west, west, north, north, and north.

Section 2: Activity 3

1. a. Textbook questions 1 and 3 of "Practice," p. 167

1. $\frac{2}{y}+\frac{4}{y}-\frac{5}{y}=\frac{2+4-5}{y}$

$=\frac{1}{y}$

Restriction: $y \neq 0$

3. $\frac{4}{x+3}+\frac{5}{x+3}=\frac{4+5}{x+3}$

$=\frac{9}{x+3}$

Restriction: $x \neq -3$

b. Textbook questions 5, 7, and 9 of "Practice," p. 167

5. $\frac{x+7}{2}+\frac{x+4}{2}=\frac{x+7+x+4}{2}$

$=\frac{2x+11}{2}$

Restrictions: none

7. $\frac{3a-1}{a}-\frac{4a+2}{a}=\frac{3a-1-(4a+2)}{a}$

$=\frac{3a-1-4a-2}{a}$

$=\frac{-a-3}{a}$

Restriction: $a \neq 0$

9. $\frac{x^2+4}{x+1}+\frac{2x^2}{x+1}=\frac{x^2+4+2x^2}{x+1}$

$=\frac{3x^2+4}{x+1}$

Restriction: $x \neq -1$

2. a. Textbook questions 15 and 17 of "Practice," p. 167

15. $\frac{2x}{2}+\frac{x}{3}=\frac{x}{1}+\frac{x}{3}$

$=\frac{3(x)}{3(1)}+\frac{x}{3}$

$=\frac{3x}{3}+\frac{x}{3}$

$=\frac{4x}{3}$

Restrictions: none

17. $\frac{x}{5}-\frac{y}{2}+\frac{7}{10}=\frac{2(x)}{2(5)}-\frac{5(y)}{5(2)}+\frac{7}{10}$

$=\frac{2x}{10}-\frac{5y}{10}+\frac{7}{10}$

$=\frac{2x-5y+7}{10}$

Restrictions: none

b. Textbook questions 19, 21, 23, and 25 of "Practice," p. 167

19. $\frac{2m+3}{2}+\frac{3m+4}{7}=\frac{7(2m+3)}{7(2)}+\frac{2(3m+4)}{2(7)}$

$=\frac{14m+21}{14}+\frac{6m+8}{14}$

$=\frac{14m+21+6m+8}{14}$

$=\frac{20m+29}{14}$ **Restrictions:** none

21. $\frac{y-5}{6}-\frac{2y-3}{4}=\frac{2(y-5)}{2(6)}-\frac{3(2y-3)}{3(4)}$

$=\frac{2y-10}{12}-\frac{6y-9}{12}$

$=\frac{2y-10-6y+9}{12}$

$=\frac{-4y-1}{12}$ **Restrictions:** none

23. $\frac{4t-1}{6}+\frac{3t+2}{2}-\frac{2t+1}{3}=\frac{4t-1}{6}+\frac{3(3t+2)}{3(2)}-\frac{2(2t+1)}{2(3)}$

$=\frac{4t-1}{6}+\frac{9t+6}{6}-\frac{4t+2}{6}$

$=\frac{4t-1+9t+6-4t-2}{6}$

$=\frac{9t+3}{6}$

$=\frac{\overset{1}{\cancel{3}}(3t+1)}{\underset{1}{\cancel{3}}(2)}$

$=\frac{3t+1}{2}$ **Restrictions:** none

Section 2: Activity 3 (continued)

25. $\frac{5x-1}{5}+1-\frac{4x-3}{6}=\frac{6(5x-1)}{6(5)}+\frac{30(1)}{30(1)}-\frac{5(4x-3)}{5(6)}$

$=\frac{30x-6}{30}+\frac{30}{30}-\frac{20x-15}{30}$

$=\frac{30x-6+30-20x+15}{30}$

$=\frac{10x+39}{30}$ **Restrictions:** none

c. Textbook question 29 of "Applications and Problem Solving," p. 168

29. a. $P=2\left(\frac{3x+1}{2}\right)+2\left(\frac{2x-1}{3}\right)$

$=3x+1+\frac{4x-2}{3}$

$=\frac{3(3x+1)}{3(1)}+\frac{4x-2}{3}$

$=\frac{9x+3+4x-2}{3}$

$=\frac{13x+1}{3}$

b. The numerator must be a multiple of 3. Assume x is a natural number.

If $x=2$, then $\frac{13(2)+1}{3}=\frac{27}{3}$

$=9$

If $x=5$, then $\frac{13(5)+1}{3}=\frac{66}{3}$

$=22$

If $x=8$, then $\frac{13(8)+1}{3}=\frac{105}{3}$

$=35$

The three smallest values of x that give whole-number values for the perimeter are 2, 5, and 8.

3. Textbook questions 9, 11, 13, and 15 of "Practice," p. 172

9. $2x = 2 \times x$
$5x = 5 \times x$

The LCD is $2 \times 5 \times x = 10x$.

$$\therefore \frac{3}{2x} + \frac{4}{5x} = \frac{5(3)}{5(2x)} + \frac{2(4)}{2(5x)}$$
$$= \frac{15}{10x} + \frac{8}{10x}$$
$$= \frac{23}{10x} \quad \textbf{Restriction: } x \neq 0$$

11. $2x^2 = 2 \times x \times x$
$3x = 3 \times x$
$x^3 = x \times x \times x$

The LCD is $2 \times 3 \times x \times x \times x = 6x^3$.

$$\therefore \frac{1}{2x^2} + \frac{3}{3x} - \frac{2}{x^3} = \frac{3x(1)}{3x(2x^2)} + \frac{2x^2(3)}{2x^2(3x)} - \frac{6(2)}{6(x^3)}$$
$$= \frac{3x}{6x^3} + \frac{6x^2}{6x^3} - \frac{12}{6x^3}$$
$$= \frac{3x + 6x^2 - 12}{6x^3}$$
$$= \frac{6x^2 + 3x - 12}{6x^3}$$
$$= \frac{\overset{1}{\cancel{3}}(2x^2 + x - 4)}{\underset{1}{\cancel{3}}(2x^3)}$$
$$= \frac{2x^2 + x - 4}{2x^3} \quad \textbf{Restriction: } x \neq 0$$

Section 2: Activity 3 (continued)

13. The LCD is x.

$$x-\frac{2}{x}+5=\frac{x(x)}{x(1)}-\frac{2}{x}+\frac{x(5)}{x(1)}$$

$$=\frac{x^2}{x}-\frac{2}{x}+\frac{5x}{x}$$

$$=\frac{x^2-2+5x}{x}$$

$$=\frac{x^2+5x-2}{x}$$ **Restriction:** $x \neq 0$

15. $3x^2 = 3 \times x \times x$

$x = x$

$5x^2 = 5 \times x \times x$

The LCD is $3\times5\times x\times x = 15x^2$.

$$\therefore \frac{4x-1}{3x^2}-\frac{2x+3}{x}+\frac{5x+2}{5x^2}=\frac{5(4x-1)}{5\left(3x^2\right)}-\frac{15x(2x+3)}{15x(x)}+\frac{3(5x+2)}{3\left(5x^2\right)}$$

$$=\frac{20x-5}{15x^2}-\frac{30x^2+45x}{15x^2}+\frac{15x+6}{15x^2}$$

$$=\frac{20x-5-30x^2-45x+15x+6}{15x^2}$$

$$=\frac{-30x^2-10x+1}{15x^2}$$ **Restriction:** $x \neq 0$

4. Textbook questions 21, 23, and 25 of "Practice," p. 172

21. $x+3=\quad x+3$
$4x+12=4\times(x+3)$

The LCD is $4(x+3)$.

$$\therefore \frac{4}{x+3}+\frac{5}{4x+12}=\frac{4(4)}{4(x+3)}+\frac{5}{4(x+3)}$$
$$=\frac{16+5}{4(x+3)}$$
$$=\frac{21}{4(x+3)} \qquad \textbf{Restriction: } x\neq -3$$

23. $t-4=\quad t-4$
$3t-12=3\times(t-4)$

The LCD is $3(t-4)$.

$$\therefore \frac{t}{t-4}-\frac{2t}{3t-12}=\frac{3(t)}{3(t-4)}-\frac{2t}{3(t-4)}$$
$$=\frac{3t-2t}{3(t-4)}$$
$$=\frac{t}{3(t-4)} \qquad \textbf{Restriction: } t\neq 4$$

25. $4y-8=4\times(y-2)$
$3y-6=\quad 3\times(y-2)$

The LCD is $4\times3\times(y-2)=12(y-2)$.

$$\therefore \frac{3}{4y-8}-\frac{2}{3y-6}=\frac{3(3)}{3(4)(y-2)}-\frac{4(2)}{4(3)(y-2)}$$
$$=\frac{9}{12(y-2)}-\frac{8}{12(y-2)}$$
$$=\frac{1}{12(y-2)} \qquad \textbf{Restriction: } y\neq 2$$

Section 2: Activity 3 (continued)

5. a. Textbook questions 27, 29, and 31 of "Practice," p. 172

27. $\frac{2}{x+1}+\frac{3}{x+2}=\frac{x+2}{x+2}\times\frac{2}{x+1}+\frac{x+1}{x+1}\times\frac{3}{x+2}$

$=\frac{2(x+2)}{(x+1)(x+2)}+\frac{3(x+1)}{(x+1)(x+2)}$

$=\frac{2x+4+3x+3}{(x+1)(x+2)}$

$=\frac{5x+7}{(x+1)(x+2)}$ **Restrictions:** $x\neq -2,\ -1$

29. $\frac{3}{x}+\frac{5}{x-1}=\frac{x-1}{x-1}\times\frac{3}{x}+\frac{x}{x}\times\frac{5}{x-1}$

$=\frac{3(x-1)}{x(x-1)}+\frac{5x}{x(x-1)}$

$=\frac{3x-3+5x}{x(x-1)}$

$=\frac{8x-3}{x(x-1)}$ **Restrictions:** $x\neq 0,\ 1$

31. $\frac{2x}{x-2}-\frac{3x}{x+2}=\frac{x+2}{x+2}\times\frac{2x}{x-2}-\frac{x-2}{x-2}\times\frac{3x}{x+2}$

$=\frac{2x(x+2)}{(x+2)(x-2)}-\frac{3x(x-2)}{(x+2)(x-2)}$

$=\frac{2x^2+4x-3x^2+6x}{(x+2)(x-2)}$

$=\frac{10x-x^2}{(x+2)(x-2)}$ **Restrictions:** $x\neq \pm 2$

b. Textbook questions 33 and 35 of "Practice," p. 172

33. $2x-2=2(x-1)=$ 2 $\times (x-1)$

$4x-8=4(x-2)=$ 2×2 $\times (x-2)$

The LCD is $4(x-1)(x-2)$.

$$\therefore \frac{1}{2x-2}+\frac{3}{4x-8}=\frac{2(x-2)}{2(x-2)}\times\frac{1}{2(x-1)}+\frac{x-1}{x-1}\times\frac{3}{4(x-2)}$$

$$=\frac{2(x-2)}{4(x-1)(x-2)}+\frac{3(x-1)}{4(x-1)(x-2)}$$

$$=\frac{2x-4+3x-3}{4(x-1)(x-2)}$$

$$=\frac{5x-7}{4(x-1)(x-2)}$$ **Restrictions:** $x \neq 1,\ 2$

35. $2s-12=2(s-6)=$ 2 $\times (s-6)$

$5s-5=5(s-1)=$ 5 $\times (s-1)$

The LCD is $10(s-1)(s-6)$.

$$\therefore \frac{4}{2s-12}-\frac{s}{5s-5}=\frac{5(s-1)}{5(s-1)}\times\frac{4}{2(s-6)}-\frac{2(s-6)}{2(s-6)}\times\frac{s}{5(s-1)}$$

$$=\frac{20(s-1)}{10(s-1)(s-6)}-\frac{2s(s-6)}{10(s-1)(s-6)}$$

$$=\frac{20s-20-2s^2+12s}{10(s-1)(s-6)}$$

$$=\frac{-2s^2+32s-20}{10(s-1)(s-6)}$$

$$=\frac{\overset{1}{\cancel{2}}\left(-s^2+16s-10\right)}{\underset{1}{\cancel{2}}(5)(s-1)(s-6)}$$

$$=\frac{-s^2+16s-10}{5(s-1)(s-6)}$$ **Restrictions:** $s \neq 1,\ 6$

Section 2: Activity 3 (continued)

6. a. Textbook questions 43, 45, and 47 of "Practice," p. 172

43. $y^2 - 16 = (y-4) \times (y+4)$

$y+4 = \quad\quad (y+4)$

The LCD is $(y-4)(y+4)$.

$$\therefore \frac{y}{y^2-16} - \frac{4}{y+4} = \frac{y}{(y-4)(y+4)} - \frac{y-4}{y-4} \times \frac{4}{y+4}$$

$$= \frac{y}{(y-4)(y+4)} - \frac{4(y-4)}{(y-4)(y+4)}$$

$$= \frac{y-4y+16}{(y-4)(y+4)}$$

$$= \frac{-3y+16}{(y-4)(y+4)}$$

Restrictions: $y \neq \pm 4$

45. $a^2 - 7a + 12 = (a-3) \times (a-4)$

$a-3 = (a-3)$

The LCD is $(a-3)(a-4)$.

$$\therefore \frac{a}{a^2-7a+12} - \frac{2a}{a-3} = \frac{a}{(a-4)(a-3)} - \frac{a-4}{a-4} \times \frac{2a}{a-3}$$

$$= \frac{a}{(a-4)(a-3)} - \frac{2a(a-4)}{(a-4)(a-3)}$$

$$= \frac{a-2a^2+8a}{(a-4)(a-3)}$$

$$= \frac{9a-2a^2}{(a-4)(a-3)}$$

Restrictions: $a \neq 3,\ 4$

47. $2n-1 = 2n-1$

$6n^2-5n+1 = (2n-1) \times (3n-1)$

The LCD is $(2n-1)(3n-1)$.

$$\therefore \frac{6}{2n-1} - \frac{3}{6n^2-5n+1} = \frac{3n-1}{3n-1} \times \frac{6}{2n-1} - \frac{3}{(2n-1)(3n-1)}$$

$$= \frac{6(3n-1)}{(2n-1)(3n-1)} - \frac{3}{(2n-1)(3n-1)}$$

$$= \frac{18n-6-3}{(2n-1)(3n-1)}$$

$$= \frac{18n-9}{(2n-1)(3n-1)}$$

$$= \frac{9\overset{1}{\cancel{(2n-1)}}}{\underset{1}{\cancel{(2n-1)}}(3n-1)}$$

$$= \frac{9}{3n-1} \quad \textbf{Restrictions: } n \neq \frac{1}{3}, \frac{1}{2}$$

b. Textbook questions 49, 51, and 53 of "Practice," p. 172

49. $x^2+4x+4 = (x+2) \times (x+2)$

$x^2-4 = (x+2) \times (x-2)$

The LCD is $(x+2)(x+2)(x-2) = (x+2)^2(x-2)$.

$$\therefore \frac{1}{x^2+4x+4} - \frac{3}{x^2-4} = \frac{x-2}{x-2} \times \frac{1}{(x+2)^2} - \frac{x+2}{x+2} \times \frac{3}{(x+2)(x-2)}$$

$$= \frac{x-2}{(x+2)^2(x-2)} - \frac{3(x+2)}{(x+2)^2(x-2)}$$

$$= \frac{x-2-3x-6}{(x+2)^2(x-2)}$$

$$= \frac{-2x-8}{(x+2)^2(x-2)} \quad \textbf{Restrictions: } x \neq \pm 2$$

Section 2: Activity 3 (continued)

51. $m^2-9m+18=(m-3)\times(m-6)$

$m^2-11m+30=(m-6)\times(m-5)$

The LCD is $(m-3)(m-6)(m-5)$.

$$\therefore \frac{4m}{m^2-9m+18}+\frac{2m}{m^2-11m+30}=\frac{m-5}{m-5}\times\frac{4m}{(m-3)(m-6)}+\frac{m-3}{m-3}\times\frac{2m}{(m-6)(m-5)}$$

$$=\frac{4m(m-5)}{(m-3)(m-6)(m-5)}+\frac{2m(m-3)}{(m-3)(m-6)(m-5)}$$

$$=\frac{4m^2-20m+2m^2-6m}{(m-3)(m-6)(m-5)}$$

$$=\frac{6m^2-26m}{(m-3)(m-6)(m-5)}$$ **Restrictions:** $m\neq 3,\ 5,\ 6$

53. $4y^2-9=(2y-3)\times(2y+3)$

$4y^2-12y+9=(2y-3)\times(2y-3)$

The LCD is $(2y-3)(2y+3)(2y-3)=(2y-3)^2(2y+3)$.

$$\therefore \frac{3y}{4y^2-9}-\frac{2y}{4y^2-12y+9}=\frac{2y-3}{2y-3}\times\frac{3y}{(2y-3)(2y+3)}-\frac{2y+3}{2y+3}\times\frac{2y}{(2y-3)^2}$$

$$=\frac{3y(2y-3)}{(2y-3)^2(2y+3)}-\frac{2y(2y+3)}{(2y-3)^2(2y+3)}$$

$$=\frac{6y^2-9y-4y^2-6y}{(2y-3)^2(2y+3)}$$

$$=\frac{2y^2-15y}{(2y-3)^2(2y+3)}$$ **Restrictions:** $y\neq\pm\frac{3}{2}$

c. Textbook questions 55, 57, 59, and 61 of "Practice," pp. 172 and 173

55. $y-1=\quad y-1$

$y^2+y-2=(y-1)\times(y+2)$

The LCD is $(y-1)(y+2)$.

$$\therefore \frac{y+1}{y-1}+\frac{y-1}{y^2+y-2}=\frac{y+2}{y+2}\times\frac{y+1}{y-1}+\frac{y-1}{(y-1)(y+2)}$$

$$=\frac{(y+1)(y+2)}{(y-1)(y+2)}+\frac{y-1}{(y-1)(y+2)}$$

$$=\frac{y^2+3y+2+y-1}{(y-1)(y+2)}$$

$$=\frac{y^2+4y+1}{(y-1)(y+2)}$$ **Restrictions:** $y\neq -2,\ 1$

57. $n^2-16=\quad (n-4)\times(n+4)$

$3n-12=3\times(n-4)$

The LCD is $3(n-4)(n+4)$.

$$\therefore \frac{n^2+4n-3}{n^2-16}+\frac{4-3n}{3n-12}=\frac{3}{3}\times\frac{n^2+4n-3}{(n-4)(n+4)}-\frac{n+4}{n+4}\times\frac{3n-4}{3(n-4)}$$

$$=\frac{3\left(n^2+4n-3\right)}{3(n-4)(n+4)}-\frac{(3n-4)(n+4)}{3(n-4)(n+4)}$$

$$=\frac{3n^2+12n-9-\left(3n^2+8n-16\right)}{3(n-4)(n+4)}$$

$$=\frac{3n^2+12n-9-3n^2-8n+16}{3(n-4)(n+4)}$$

$$=\frac{4n+7}{3(n-4)(n+4)}$$ **Restrictions:** $n\neq\pm 4$

Section 2: Activity 3 (continued)

59. $a^2-1=(a+1)\quad\quad\times(a-1)$

$a^2+2a+1=(a+1)\times(a+1)\quad\quad$

The LCD is $(a+1)(a+1)(a-1)=(a+1)^2(a-1)$.

$$\therefore\ \frac{a+2}{a^2-1}-\frac{a-1}{a^2+2a+1}=\frac{a+1}{a+1}\times\frac{a+2}{(a+1)(a-1)}-\frac{a-1}{a-1}\times\frac{a-1}{(a+1)^2}$$

$$=\frac{(a+2)(a+1)}{(a+1)^2(a-1)}-\frac{(a-1)(a-1)}{(a+1)^2(a-1)}$$

$$=\frac{a^2+3a+2-\left(a^2-2a+1\right)}{(a+1)^2(a-1)}$$

$$=\frac{a^2+3a+2-a^2+2a-1}{(a+1)^2(a-1)}$$

$$=\frac{5a+1}{(a+1)^2(a-1)}$$ **Restrictions:** $a\neq\pm1$

61. $2x^2+3x+1=(x+1)\times(2x+1)$

$3x^2+4x+1=(x+1)\quad\quad\times(3x+1)$

The LCD is $(x+1)(2x+1)(3x+1)$.

$$\therefore\ \frac{2x-1}{2x^2+3x+1}+\frac{2x+1}{3x^2+4x+1}=\frac{3x+1}{3x+1}\times\frac{2x-1}{(x+1)(2x+1)}+\frac{2x+1}{2x+1}\times\frac{2x+1}{(x+1)(3x+1)}$$

$$=\frac{(2x-1)(3x+1)}{(x+1)(2x+1)(3x+1)}+\frac{(2x+1)(2x+1)}{(x+1)(2x+1)(3x+1)}$$

$$=\frac{6x^2-x-1+4x^2+4x+1}{(x+1)(2x+1)(3x+1)}$$

$$=\frac{10x^2+3x}{(x+1)(2x+1)(3x+1)}$$ **Restrictions:** $x\neq-1,\ -\frac{1}{2},\ -\frac{1}{3}$

7. Textbook questions 1 to 6 of "Mental Math: Adding Using Compatible Numbers," p. 147

1. $47+34=81$

2. $69+53=122$

3. $34+78=112$

4. $127+69=196$

5. $114+167=281$

6. $235+148=383$

8. Textbook questions 1 and 2 of "LOGIC POWER," p. 173

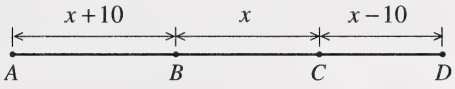
1. There are 28 cubes in the stack.

2. a. There are 8 cubes with only 1 yellow face.

b. There are 11 cubes with only 2 yellow faces.

Section 2: Activity 4

1. a.



b. **Guess 1:** Try $x=100$.
Does $110+100+90=390$? No.

Guess 2: Try $x=120$.
Does $130+120+110=390$? No.

Guess 3: Try $x=140$.
Does $150+140+130=390$? No.

Guess 4: Try $x=130$.
Does $140+130+120=390$? Yes.

The distance from B to C is 130 km.

c.
$$(x+10)+x+(x-10)=390$$
$$x+10+x+x-10=390$$
$$3x=390$$
$$x=130$$

The distance from B to C is 130 km.

Section 2: Activity 4 (continued)

2. Textbook questions 1 to 4 of "Inquire," p. 182

1.
$$0.05x = 0.8$$
$$\frac{0.05x}{0.05} = \frac{0.8}{0.05}$$
$$x = 16$$

2.
$$0.05x = 0.8$$
$$100(0.05x) = 100(0.8)$$
$$5x = 80$$
$$\frac{5x}{5} = \frac{80}{5}$$
$$x = 16$$

3.
$$0.05x = 0.8$$
$$\frac{x}{20} = \frac{4}{5}$$
$$20\left(\frac{x}{20}\right) = 20\left(\frac{4}{5}\right)$$
$$x = \frac{80}{5}$$
$$= 16$$

4. The maximum height is 16 m.

3. a. Textbook questions 11, 13, and 15 of "Practice," p. 185

11. Method 1: Keeping the Decimals

$$m + 4.6 = 5.9$$
$$m + 4.6 - 4.6 = 5.9 - 4.6$$
$$m = 1.3$$

Restrictions: none

Method 2: Clearing the Decimals

$$m + 4.6 = 5.9$$
$$10m + 46 = 59$$
$$10m = 13$$
$$m = 1.3$$

Restrictions: none

Check

LS	RS
$m + 4.6$	5.9
$= 1.3 + 4.6$	
$= 5.9$	

LS = RS

13. Method 1: Keeping the Decimals

$$0.5x = 1.8$$
$$\frac{0.5x}{0.5} = \frac{1.8}{0.5}$$
$$x = 3.6$$

Restrictions: none

Check

LS	RS
$0.5x$	1.8
$=0.5(3.6)$	
$=1.8$	

LS = RS

Method 2: Clearing the Decimals

$$0.5x = 1.8$$
$$5x = 18$$
$$x = \frac{18}{5}$$
$$x = 3.6$$

Restrictions: none

15. Method 1: Keeping the Decimals

$$2e + 1.2 = 2.8$$
$$2e + 1.2 - 1.2 = 2.8 - 1.2$$
$$2e = 1.6$$
$$\frac{2e}{2} = \frac{1.6}{2}$$
$$e = 0.8$$

Restrictions: none

Check

LS	RS
$2e+1.2$	2.8
$2(0.8)+1.2$	
$=1.6+1.2$	
$=2.8$	

LS = RS

Method 2: Clearing the Decimals

$$2e + 1.2 = 2.8$$
$$20e + 12 = 28$$
$$20e + 12 - 12 = 28 - 12$$
$$20e = 16$$
$$\frac{20e}{20} = \frac{16}{20}$$
$$e = 0.8$$

Restrictions: none

Section 2: Activity 4 (continued)

b. Textbook questions 17 and 19 of "Practice," p. 185

17. Method 1: Keeping the Decimals

$$2(a+1.5)=4.2$$
$$2a+3=4.2$$
$$2a=1.2$$
$$a=\frac{1.2}{2}$$
$$=0.6$$

Restrictions: none

Method 2: Clearing the Decimals

$$2(a+1.5)=4.2$$
$$2a+3=4.2$$
$$20a+30=42$$
$$20a=12$$
$$a=\frac{12}{20}$$
$$=0.6$$

Restrictions: none

19. Method 1: Keeping the Decimals

$$12.5=6.5+2.5(a-1)$$
$$12.5=6.5+2.5a-2.5$$
$$12.5=4+2.5a$$
$$8.5=2.5a$$
$$2.5a=8.5$$
$$a=\frac{8.5}{2.5}$$
$$=3.4$$

Restrictions: none

Method 2: Clearing the Decimals

$$12.5=6.5+2.5(a-1)$$
$$125=65+25(a-1)$$
$$125=65+25a-25$$
$$125=40+25a$$
$$85=25a$$
$$25a=85$$
$$a=\frac{85}{25}$$
$$a=3.4$$

Restrictions: none

4. a. Textbook questions 5, 7, and 9 of "Practice," p. 185

5.

$$\frac{x}{4}+6=8$$
$$4\left(\frac{x}{4}+6\right)=4(8)$$
$$x+24=32$$
$$x=8$$

Restrictions: none

Check

LS	RS
$\frac{x}{4}+6$	8
$=\frac{8}{4}+6$	
$=2+6$	
$=8$	

LS = RS

7. $-2=\frac{1}{5}x-1$

$$\frac{1}{5}x-1=-2$$
$$5\left(\frac{1}{5}x-1\right)=5(-2)$$
$$x-5=-10$$
$$x=-5$$

Restrictions: none

Check

LS	RS
-2	$\frac{1}{5}x-1$
	$=\frac{1}{5}(-5)-1$
	$=-1-1$
	$=-2$

LS = RS

9. $2=3+\frac{2x}{5}$

$$3+\frac{2x}{5}=2$$
$$5\left(3+\frac{2x}{5}\right)=5(2)$$
$$15+2x=10$$
$$2x=-5$$
$$x=-\frac{5}{2}$$

Restrictions: none

Check

LS	RS
2	$3+\frac{2x}{5}$
	$=3+\frac{2\left(-\frac{5}{2}\right)}{5}$
	$=3+\frac{(-5)}{5}$
	$=3+(-1)$
	$=2$

LS = RS

b. Textbook questions 31, 33, 35, and 37 of "Practice," p. 185

31. $\frac{3}{4}m-2=\frac{m}{4}-\frac{1}{4}$

$$4\left(\frac{3}{4}m-2\right)=4\left(\frac{m}{4}-\frac{1}{4}\right)$$
$$3m-8=m-1$$
$$3m=m+7$$
$$2m=7$$
$$m=\frac{7}{2}$$

Restrictions: none

33. $\frac{3y}{5}+2=4-\frac{3y}{5}$

$$5\left(\frac{3y}{5}+2\right)=5\left(4-\frac{3y}{5}\right)$$
$$3y+10=20-3y$$
$$3y=10-3y$$
$$6y=10$$
$$y=\frac{10}{6}$$
$$=\frac{5}{3}$$

Restrictions: none

Section 2: Activity 4 (continued)

35. $\dfrac{x+1}{4}+\dfrac{x+2}{2}=1$

$4\left(\dfrac{x+1}{4}+\dfrac{x+2}{2}\right)=4(1)$ LCD = 4

$x+1+2(x+2)=4$

$x+1+2x+4=4$

$3x+5=4$

$3x=-1$

$x=-\dfrac{1}{3}$

Restrictions: none

37. $x-\dfrac{x+2}{2}=\dfrac{x}{3}+1$

$6\left(x-\dfrac{x+2}{2}\right)=6\left(\dfrac{x}{3}+1\right)$ LCD = 6

$6x-3(x+2)=2x+6$

$6x-3x-6=2x+6$

$3x-6=2x+6$

$3x=2x+12$

$x=12$

Restrictions: none

c. Textbook questions 39, 41, and 43 of "Practice," p. 185

39. $-\dfrac{4}{n}=2$

$n\left(-\dfrac{4}{n}\right)=n(2)$

$-4=2n$

$2n=-4$

$n=-2$

Restriction: $n\neq 0$

41. $\dfrac{1}{5x}=\dfrac{1}{10}$

$10x\left(\dfrac{1}{5x}\right)=10x\left(\dfrac{1}{10}\right)$ LCD = $10x$

$2=x$

$x=2$

Restriction: $x\neq 0$

43. $\dfrac{-5}{2x} = -\dfrac{1}{4}$

$4x\left(\dfrac{-5}{2x}\right) = 4x\left(-\dfrac{1}{4}\right)$ LCD $= 4x$

$-10 = -x$

$\dfrac{-10}{-1} = \dfrac{-x}{-1}$

$10 = x$

$x = 10$

Restriction: $x \neq 0$

d. Textbook questions 45, 47, and 49 of "Practice," p. 185

45. $4 + \dfrac{1}{2x} = 5$

$2x\left(4 + \dfrac{1}{2x}\right) = 2x(5)$

$8x + 1 = 10x$

$8x = 10x - 1$

$-2x = -1$

$x = \dfrac{1}{2}$

Restriction: $x \neq 0$

47. $\dfrac{1}{d} - 2 = \dfrac{3}{d}$

$d\left(\dfrac{1}{d} - 2\right) = d\left(\dfrac{3}{d}\right)$

$1 - 2d = 3$

$-2d = 2$

$d = -1$

Restriction: $d \neq 0$

49. $0 = \dfrac{2}{3} + \dfrac{3}{x}$

$3x(0) = 3x\left(\dfrac{2}{3} + \dfrac{3}{x}\right)$ LCD $= 3x$

$0 = 2x + 9$

$2x + 9 = 0$

$2x = -9$

$x = -\dfrac{9}{2}$

Restriction: $x \neq 0$

Section 2: Activity 4 (continued)

e. Textbook questions 65, 67, and 69 of "Practice," p. 185

65.

$$\frac{4}{2x-1}=\frac{1}{x-2}$$

$$(2x-1)(x-2)\left(\frac{4}{2x-1}\right)=(2x-1)(x-2)\left(\frac{1}{x-2}\right) \qquad \text{LCD}=(2x-1)(x-2)$$

$$4(x-2)=2x-1$$

$$4x-8=2x-1$$

$$4x=2x+7$$

$$2x=7$$

$$x=\frac{7}{2} \qquad \textbf{Restrictions: } x\neq\frac{1}{2}, 2$$

67.

$$\frac{1}{2w+3}+\frac{3}{2w+3}=2$$

$$\frac{4}{2w+3}=2$$

$$(2w+3)\left(\frac{4}{2w+3}\right)=(2w+3)(2)$$

$$4=4w+6$$

$$-2=4w$$

$$4w=-2$$

$$w=\frac{-2}{4}$$

$$=-\frac{1}{2} \qquad \textbf{Restriction: } w\neq-\frac{3}{2}$$

69.

$$\frac{-1}{2m-1}=2+\frac{5}{2m-1}$$

$$(2m-1)\left(\frac{-1}{2m-1}\right)=(2m-1)\left(2+\frac{5}{2m-1}\right)$$

$$-1=2(2m-1)+5$$

$$-1=4m-2+5$$

$$-1=4m+3$$

$$-4=4m$$

$$4m=-4$$

$$m=-1 \qquad \textbf{Restriction: } m\neq\frac{1}{2}$$

f. Textbook questions 70, 71, 73, 75, and 78 of "Applications and Problem Solving," pp. 185 and 186

70. $\frac{m}{2}+6=156$

$2\left(\frac{m}{2}+6\right)=2(156)$

$m+12=312$

$m=300$

The average mass of a human heart is 300 g.

71. $2-\frac{v}{2}=-230$

$2\left(2-\frac{v}{2}\right)=2(-230)$

$4-v=-460$

$-v=-464$

$v=464$

The lowest temperature on Venus is 464°C.

73. $\frac{x+8}{2}=-18$

$2\left(\frac{x+8}{2}\right)=2(-18)$

$x+8=-36$

$x=-44$

The lowest January temperature recorded in Edmonton was $-44\,°C$.

75. $\frac{s}{5}+10=\frac{s-2}{3}$

$15\left(\frac{s}{5}+10\right)=15\left(\frac{s-2}{3}\right)$ LCD = 15

$3s+150=5s-10$

$3s=5s-160$

$-2s=-160$

$s=80$

The top speed of an ostrich is 80 km/h.

78. a. Each side of the equation is the area of the rectangle divided by its length, which gives you the width of the rectangle.

b. $\frac{30}{\ell+5}=\frac{20}{\ell}$

$\ell(\ell+5)\times\left(\frac{30}{\ell+5}\right)=\ell(\ell+5)\times\left(\frac{20}{\ell}\right)$ LCD = $\ell(\ell+5)$

$30\ell=20(\ell+5)$

$30\ell=20\ell+100$

$10\ell=100$

$\ell=10$

The length of the smaller rectangle is 10 cm. The length of the larger rectangle is $10+5=15$ cm.

c. The width of each rectangle is $\frac{20}{10}=2$ cm.

Section 2: Activity 4 (continued)

5. Textbook questions 13, 14, 16, 18, 21, and 22 of "Applications and Problem Solving," pp. 190 and 191

13. Let w represent the number of weeks Roma skated.

$$\therefore\ 175+\frac{4.5}{2}w=256$$

$$2\left(175+\frac{4.5}{2}w\right)=2(256)$$

$$350+4.5w=512$$

$$4.5w=162$$

$$w=36$$

Roma skated for 36 weeks that year.

14. Let d represent the driving distance from Moose Jaw to Medicine Hat.

$$\therefore\ \frac{d}{5}-4+d=470$$

$$5\left(\frac{d}{5}-4+d\right)=5(470)$$

$$d-20+5d=2350$$

$$6d-20=2350$$

$$6d=2370$$

$$d=395$$

It is 395 km from Moose Jaw to Medicine Hat.

16. Let m represent the number of days it takes Mars to orbit the Sun.

$$\therefore\ \frac{1}{3}m-4=225$$

$$3\left(\frac{1}{3}m-4\right)=3(225)$$

$$m-12=675$$

$$m=687$$

Mars takes 687 days to orbit the Sun.

18. Let w represent the area of Lake Winnipeg (in square kilometres).

$$\therefore\ w+\frac{w}{4}-700=29\,800$$
$$w+\frac{w}{4}=30\,500$$
$$4\left(w+\frac{w}{4}\right)=4(30\,500)$$
$$4w+w=122\,000$$
$$5w=122\,000$$
$$w=24\,400$$

The area of Lake Winnipeg is $24\,400\ \text{km}^2$.

Now, calculate the area of Lake Winnipegosis.

$$\frac{24\,400}{4}-700=6100-700$$
$$=5400$$

The area of Lake Winnipegosis is $5400\ \text{km}^2$.

21. a. Let d represent the distance (in kilometres) from Raoul's home to his sister's home.

$$\therefore\ \frac{d}{12}+\frac{d}{8}=1$$
$$24\left(\frac{d}{12}+\frac{d}{8}\right)=24(1)$$ LCD = 24
$$2d+3d=24$$
$$5d=24$$
$$d=4.8$$

It is 4.8 km from Raoul's home to his sister's home.

b. Let t represent the time (in hours) Raoul spent jogging.

$$\therefore\ t=\frac{4.8}{12}$$
$$=0.4$$

Raoul spent $0.4\times60=24$ min jogging.

Section 2: Activity 4 (continued)

22. Let s represent the speed of the boat (in kilometres per hour).

$$\therefore \frac{120}{s+10} = \frac{80}{s}$$

$$s(s+10)\left(\frac{120}{s+10}\right) = s(s+10)\left(\frac{80}{s}\right) \qquad \text{LCD} = s(s+10)$$

$$120s = 80(s+10)$$

$$120s = 80s + 800$$

$$40s = 800$$

$$s = 20$$

The speed of the boat is 20 km/h, and the speed of the hydrofoil is $20+10=30$ km/h.

6. a. Textbook questions a. and b. of "Explore: Solve the Formula," p. 192

a.

$$\ell = e + \frac{g}{5}$$

$$5(\ell) = 5\left(e + \frac{g}{5}\right)$$

$$5\ell = 5e + g$$

$$5\ell - 5e = g$$

$$g = 5\ell - 5e$$

$$g = 5(\ell - e)$$

Either formula is acceptable.

b.

$$\ell = e + \frac{g}{5}$$

$$5(\ell) = 5\left(e + \frac{g}{5}\right)$$

$$5\ell = 5e + g$$

$$5\ell - g = 5e$$

$$5e = 5\ell - g$$

$$e = \frac{5\ell - g}{5}$$

b. Textbook questions 1 to 3 of "Inquire," p. 192

1.

$$\ell = e + \frac{g}{5}$$

$$= 47.8 + \frac{20}{5} \qquad 35 - 15 = 20$$

$$= 47.8 + 4$$

$$= 51.8$$

Melina's ears should be 51.8 mm long when she is 35 years old.

2. $g = 5(\ell - e)$

$= 5(64 - 55)$

$= 5(9)$

$= 45$

Rory will be about $45 + 15 = 60$ years old.

3. $e = \dfrac{5\ell - g}{5}$

$= \dfrac{5(54) - 25}{5}$ $\quad 40 - 15 = 25$

$= \dfrac{270 - 25}{5}$

$= \dfrac{245}{5}$

$= 49$

Kathy's ears were 49 mm long when she was 15 years old.

c. Textbook questions 1, 3, and 5 of "Practice," p. 193

1. $2ax = b$

$\dfrac{2ax}{2a} = \dfrac{b}{2a}$

$x = \dfrac{b}{2a}$

Restriction: $a \neq 0$

3. $ax - cx = 4 + b$

$x(a - c) = 4 + b$

$\dfrac{x\cancel{(a-c)}}{\cancel{(a-c)}} = \dfrac{4+b}{a-c}$

$x = \dfrac{4+b}{a-c}$

Restriction: $a \neq c$

5. $2(x + 3) = x - b$

$2x + 6 = x - b$

$2x = x - b - 6$

$x = -b - 6$

Restrictions: none

Section 2: Activity 4 (continued)

d. Textbook questions 7, 9, and 11 of "Practice," p. 193

7. $d = st$

$$\frac{d}{t} = \frac{st}{t}$$

$$\frac{d}{t} = s$$

$$s = \frac{d}{t}$$

Restriction: $t \neq 0$

9. $Ax + By + C = 0$

$$By = -Ax - C$$

$$\frac{By}{B} = \frac{-Ax - C}{B}$$

$$y = \frac{-Ax - C}{B}$$

Restriction: $B \neq 0$

11.

$$A = \frac{h}{2}(a + b)$$

$$2A = h(a + b)$$

$$\frac{2A}{h} = a + b$$

$$\frac{2A}{h} - b = a$$

$$a = \frac{2A}{h} - b$$

Restriction: $h \neq 0$

e. Textbook questions 15 and 16.a. of "Applications and Problem Solving," p. 193

15. Step 1: Solve the formula for a.

$$r = 176 - \frac{4}{5}a$$

$$5(r) = 5\left(176 - \frac{4}{5}a\right)$$

$$5r = 880 - 4a$$

$$5r + 4a = 880$$

$$4a = 880 - 5r$$

$$a = \frac{880 - 5r}{4}$$

Step 2: Find Vlad's age.

$$\begin{aligned} a &= \frac{880-5r}{4} \\ &= \frac{880-5(140)}{4} \\ &= \frac{880-700}{4} \\ &= \frac{180}{4} \\ &= 45 \end{aligned}$$

Vlad is 45 years old.

16. a. Step 1: Solve for C.

$$E = 16.8(P+C+2.25F)$$

$$\frac{E}{16.8} = P+C+2.25F$$

$$\frac{E}{16.8} - 2.25F = P+C$$

$$\frac{E}{16.8} - 2.25F - P = C$$

$$C = \frac{E}{16.8} - 2.25F - P$$

Step 2: Find the numbers of grams of carbohydrates in a bagel.

$$\begin{aligned} C &= \frac{E}{16.8} - 2.25F - P \\ &= \frac{840}{16.8} - 2.25(2) - 7 \\ &= 50 - 4.50 - 7 \\ &= 38.5 \end{aligned}$$

A bagel contains 38.5 g of carbohydrates.

Section 2: Activity 4 (continued)

7. Textbook questions 1 and 2 of "Data Bank," p. 203

1. a. Edmonton–Vancouver–Seattle–Portland

$$1245 + 230 + 280 = 1755 \text{ km}$$

Edmonton–Calgary–Spokane–Portland

$$295 + 720 + 560 = 1575 \text{ km}$$

The shorter route from Edmonton to Portland is through Calgary and Spokane.

b. First, find the difference in the distances.

$$1755 - 1575 = 180$$

Now, determine the amount of time saved.

$$\frac{180 \text{ km}}{90 \text{ km/h}} = 2 \text{ h}$$

You would save 2 h by taking the shorter route.

2. a. 28°C
b. 30°C

Section 2: Follow-up Activities

Extra Help

Textbook questions 9, 11, 13, and 15 of "Review," p. 200

9. $\dfrac{3x}{3x+9} = \dfrac{\overset{1}{\cancel{3}x}}{\underset{1}{\cancel{3}(x+3)}}$

$$= \frac{x}{x+3}$$

Restriction: $x \neq -3$

11. $\dfrac{5x-5y}{7x-7y} = \dfrac{5\overset{1}{\cancel{(x-y)}}}{7\underset{1}{\cancel{(x-y)}}}$

$$= \frac{5}{7}$$

Restriction: $x \neq y$

13. $\dfrac{t-2}{t^2-3t+2} = \dfrac{\overset{1}{\cancel{(t-2)}}}{(t-1)\underset{1}{\cancel{(t-2)}}}$

$= \dfrac{1}{t-1}$

Restrictions: $t \neq 1,\ 2$

15. $\dfrac{y^2-9}{y^2+y-12} = \dfrac{(y+3)\overset{1}{\cancel{(y-3)}}}{\underset{1}{\cancel{(y-3)}}(y+4)}$

$= \dfrac{y+3}{y+4}$

Restrictions: $y \neq -4,\ 3$

Enrichment

1. Textbook questions 49, 51, and 53 of "Practice," p. 164

49. $\dfrac{5x}{\frac{x}{2}} = 5x \div \dfrac{x}{2}$

$= 5x \times \dfrac{2}{x}$

$= \dfrac{5\overset{1}{\cancel{(x)}}(2)}{\underset{1}{\cancel{(x)}}}$

$= 10$

Restriction: $x \neq 0$

51. $\dfrac{\frac{2t-4}{3t+5}}{\frac{4t-8}{2t-1}} = \dfrac{2t-4}{3t+5} \div \dfrac{4t-8}{2t-1}$

$= \dfrac{2t-4}{3t+5} \times \dfrac{2t-1}{4t-8}$

$= \dfrac{\overset{1}{2}\cancel{(t-2)} \times (2t-1)}{(3t+5) \times \underset{2}{\cancel{(4)}}\underset{1}{\cancel{(t-2)}}}$

$= \dfrac{2t-1}{2(3t+5)}$

Restrictions: $t \neq \dfrac{-5}{3},\ \dfrac{1}{2},\ 2$

53. $\dfrac{\frac{3n+6}{2n+2}}{\frac{n+2}{n^2-1}} = \dfrac{3n+6}{2n+2} \div \dfrac{n+2}{n^2-1}$

$= \dfrac{3n+6}{2n+2} \times \dfrac{n^2-1}{n+2}$

$= \dfrac{3\overset{1}{\cancel{(n+2)}} \times \overset{1}{\cancel{(n+1)}}(n-1)}{2\underset{1}{\cancel{(n+1)}} \times \underset{1}{\cancel{(n+2)}}}$

$= \dfrac{3(n-1)}{2}$

$= \dfrac{3n-3}{2}$

Restrictions: $n \neq \pm 1,\ -2$

Enrichment (continued)

2. Textbook question 66 of "Applications and Problem Solving," p. 173

66. a. $\dfrac{2+\frac{1}{x}}{2-\frac{1}{x}} = \dfrac{\frac{2x}{x}+\frac{1}{x}}{\frac{2x}{x}-\frac{1}{x}}$

$$= \frac{\frac{2x+1}{x}}{\frac{2x-1}{x}}$$

$$= \frac{2x+1}{x} \div \frac{2x-1}{x}$$

$$= \frac{2x+1}{x} \times \frac{x}{2x-1}$$

$$= \frac{(2x+1)(\cancel{x})}{(\cancel{x})(2x-1)}$$

$$= \frac{2x+1}{2x-1} \qquad \textbf{Restrictions: } x \neq 0,\ \frac{1}{2}$$

b. $\dfrac{y+\frac{1}{4}}{y-\frac{1}{2}} = \dfrac{\frac{4y}{4}+\frac{1}{4}}{\frac{2y}{2}-\frac{1}{2}}$

$$= \frac{\frac{4y+1}{4}}{\frac{2y-1}{2}}$$

$$= \frac{4y+1}{4} \div \frac{2y-1}{2}$$

$$= \frac{4y+1}{4} \times \frac{2}{2y-1}$$

$$= \frac{(4y+1)(\cancel{2})}{\cancel{4}(2y-1)}$$

$$= \frac{4y+1}{4y-2} \qquad \textbf{Restriction: } y \neq \frac{1}{2}$$

c. $\dfrac{1-\frac{1}{m^2}}{2-\frac{2}{m}} = \dfrac{\frac{m^2}{m^2}-\frac{1}{m^2}}{\frac{2m}{m}-\frac{2}{m}}$

$$= \frac{\frac{m^2-1}{m^2}}{\frac{2m-2}{m}}$$

$$= \frac{m^2-1}{m^2} \div \frac{2m-2}{m}$$

$$= \frac{m^2-1}{m^2} \times \frac{m}{2m-2}$$

$$= \frac{(m+1)\overset{1}{\cancel{(m-1)}} \times \overset{1}{\cancel{(m)}}}{(m)\underset{1}{\cancel{(m)}} \times (2)\underset{1}{\cancel{(m-1)}}}$$

$$= \frac{m+1}{2m} \qquad \textbf{Restrictions: } m \neq 0,\ 1$$

d. $\dfrac{\frac{t}{2}+1}{\frac{1}{4}+\frac{t+1}{t^2}} = \dfrac{\frac{t}{2}+\frac{2}{2}}{\frac{t^2}{4t^2}+\frac{4t+4}{4t^2}}$

$$= \frac{\frac{t+2}{2}}{\frac{t^2+4t+4}{4t^2}}$$

$$= \frac{t+2}{2} \div \frac{t^2+4t+4}{4t^2}$$

$$= \frac{t+2}{2} \times \frac{4t^2}{t^2+4t+4}$$

$$= \frac{\overset{1}{\cancel{(t+2)}} \times \overset{1}{\cancel{(2)}}(2)(t)(t)}{\underset{1}{\cancel{2}} \times \underset{1}{\cancel{(t+2)}}(t+2)}$$

$$= \frac{2t^2}{t+2} \qquad \textbf{Restrictions: } t \neq -2,\ 0$$

Enrichment (continued)

e. $\dfrac{\frac{n^2+9}{2}+3n}{n^2-9}=\dfrac{\frac{n^2+9}{2}+\frac{6n}{2}}{n^2-9}$

$$=\frac{\frac{n^2+6n+9}{2}}{n^2-9}$$

$$=\frac{n^2+6n+9}{2}\div\left(n^2-9\right)$$

$$=\frac{n^2+6n+9}{2}\times\frac{1}{n^2-9}$$

$$=\frac{(n+3)\overset{1}{\cancel{(n+3)}}}{2\times\underset{1}{\cancel{(n+3)}}(n-3)}$$

$$=\frac{n+3}{2n-6}\qquad \textbf{Restrictions: } n\neq\pm3$$

f. $\dfrac{x^2-\frac{(5x+3)}{2}}{x^2-\frac{(8x+3)}{3}}=\dfrac{\frac{2x^2}{2}-\frac{(5x+3)}{2}}{\frac{3x^2}{3}-\frac{(8x+3)}{3}}$

$$=\frac{\frac{2x^2-5x-3}{2}}{\frac{3x^2-8x-3}{3}}$$

$$=\frac{2x^2-5x-3}{2}\div\frac{3x^2-8x-3}{3}$$

$$=\frac{2x^2-5x-3}{2}\times\frac{3}{3x^2-8x-3}$$

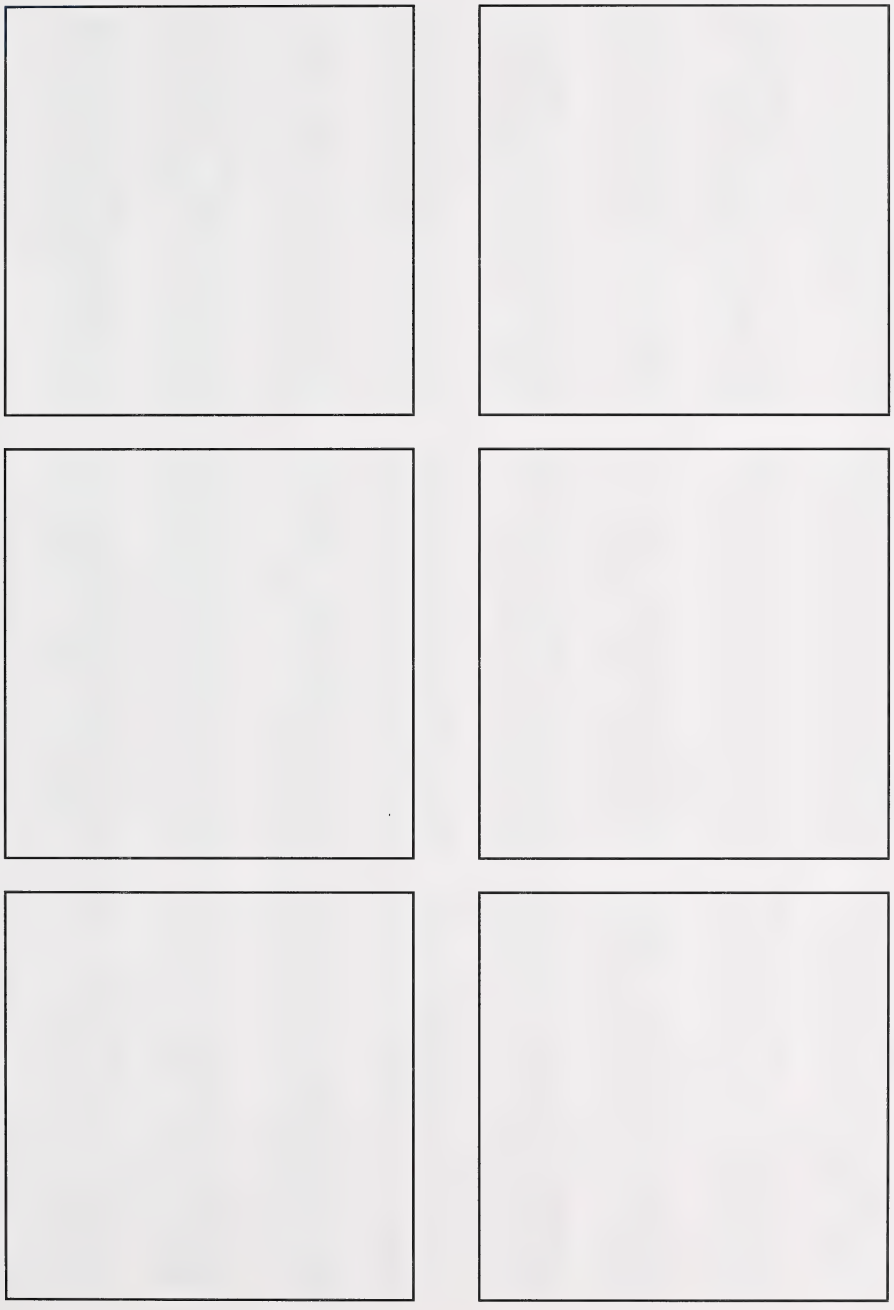
$$=\frac{(2x+1)\overset{1}{\cancel{(x-3)}}\times(3)}{2\times(3x+1)\underset{1}{\cancel{(x-3)}}}$$

$$=\frac{6x+3}{6x+2}\qquad \textbf{Restrictions: } x\neq-\frac{1}{3},\ 3$$

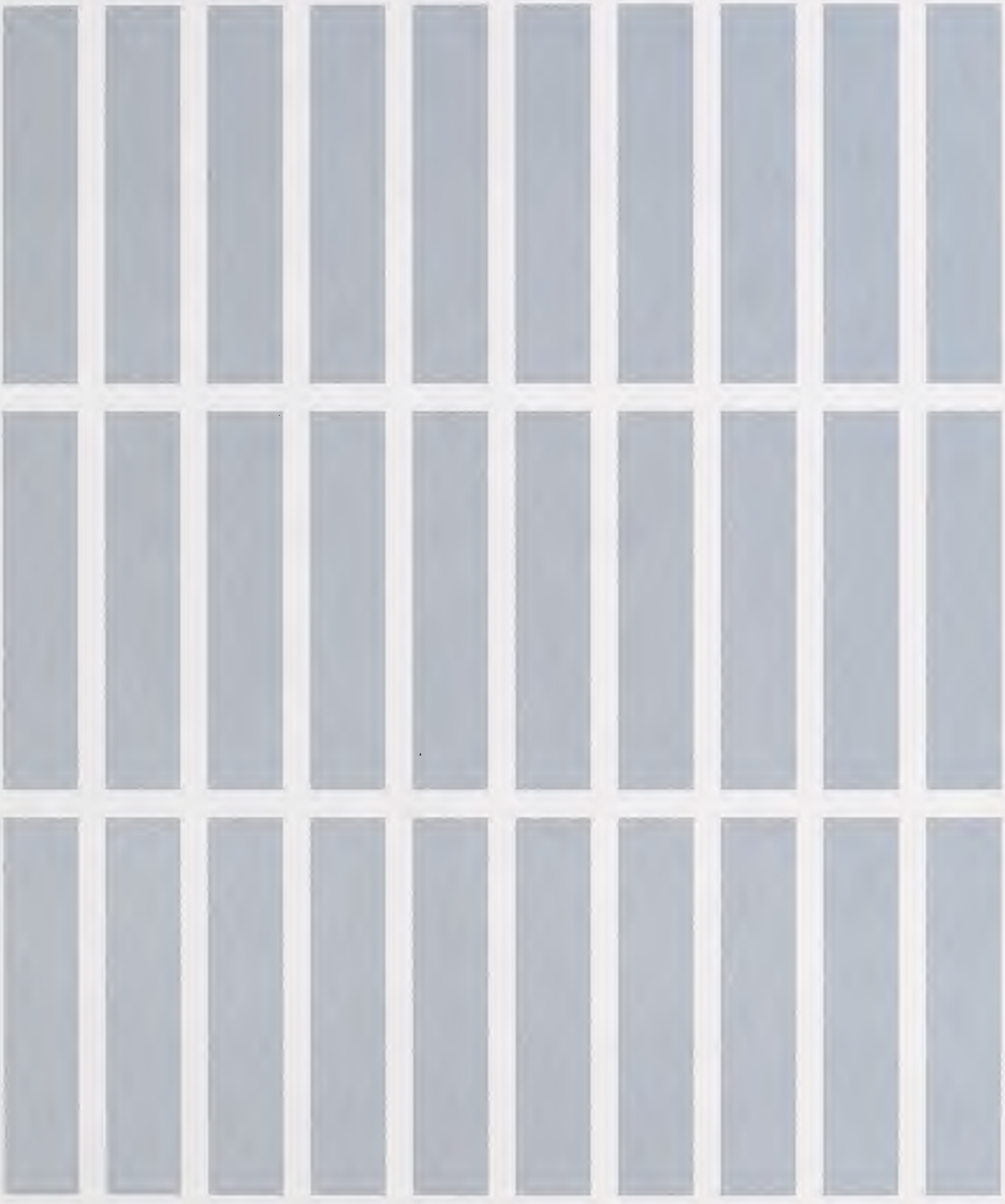
Cut-out Learning Aids

x^2-Tiles

$-x^2$-Tiles

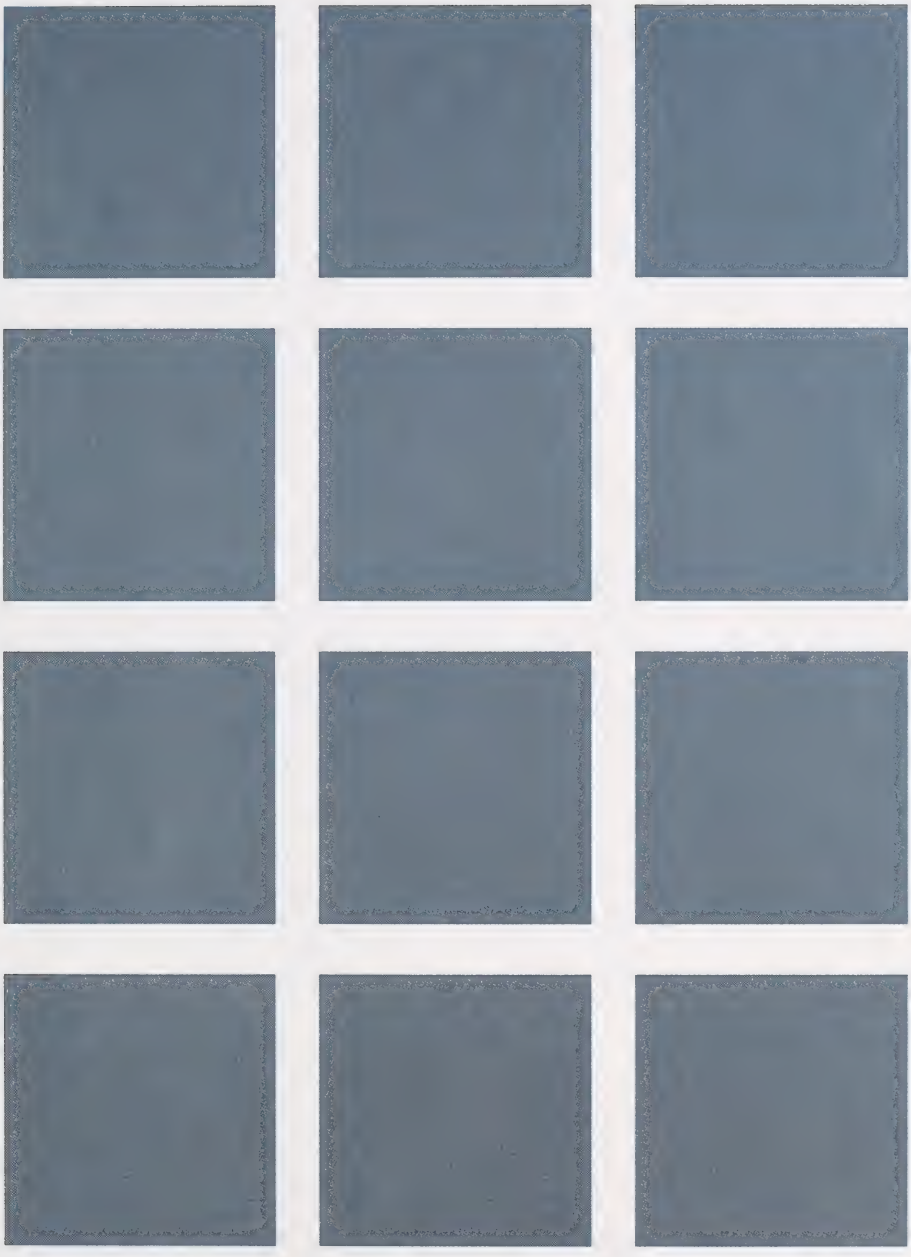


x-Tiles

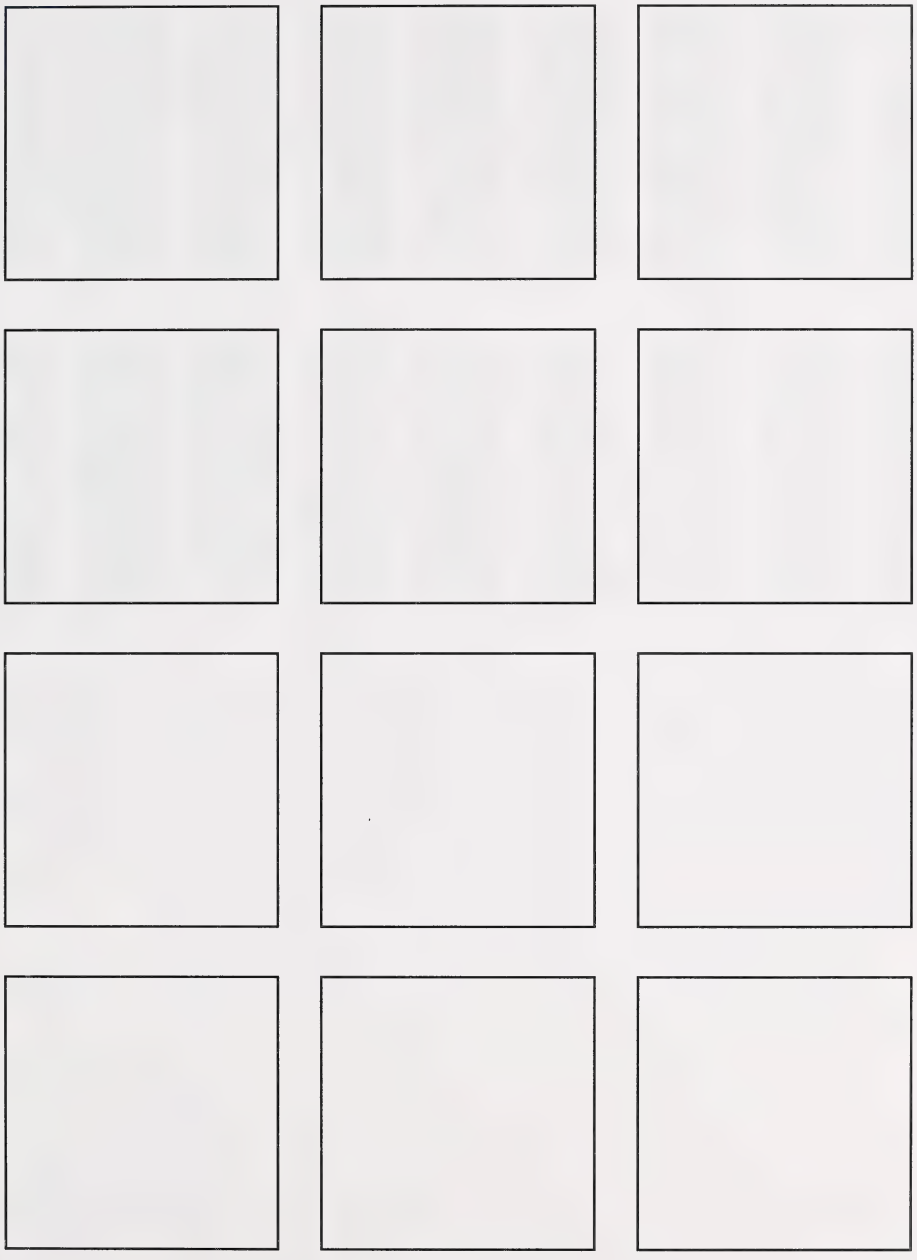


$-x$-Tiles

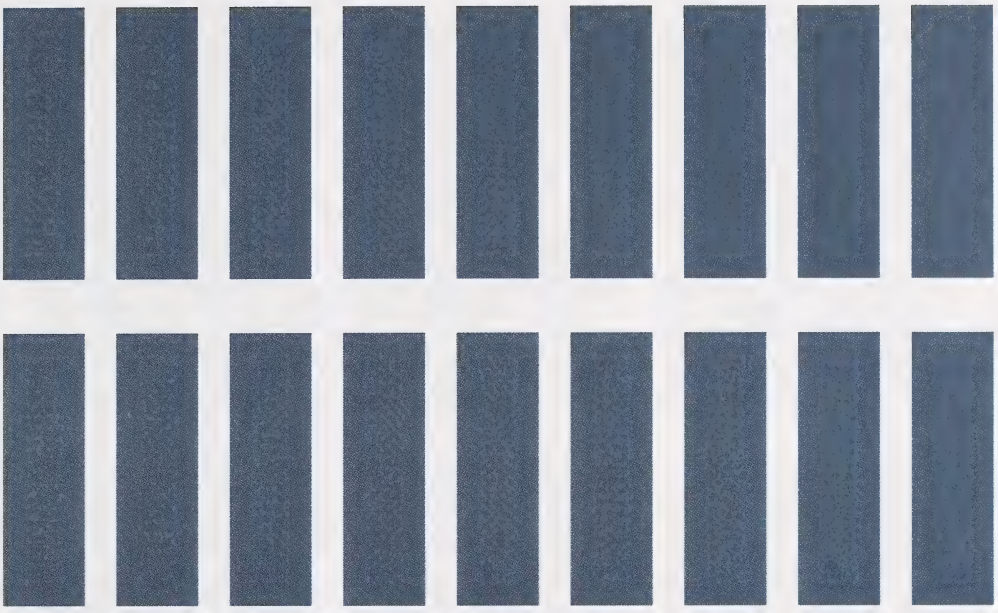
y^2-Tiles



$-y^2$-Tiles



y-Tiles



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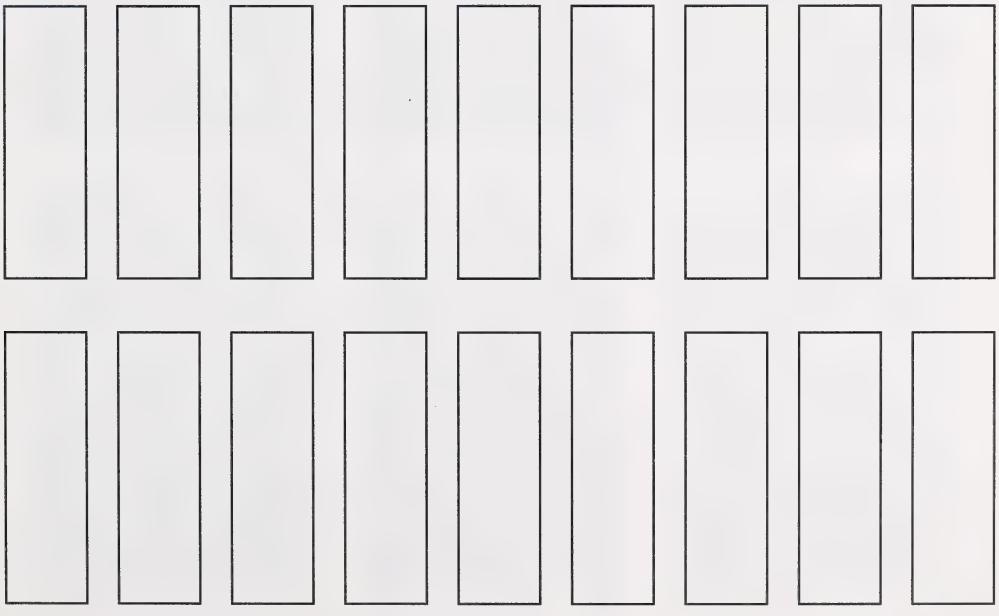
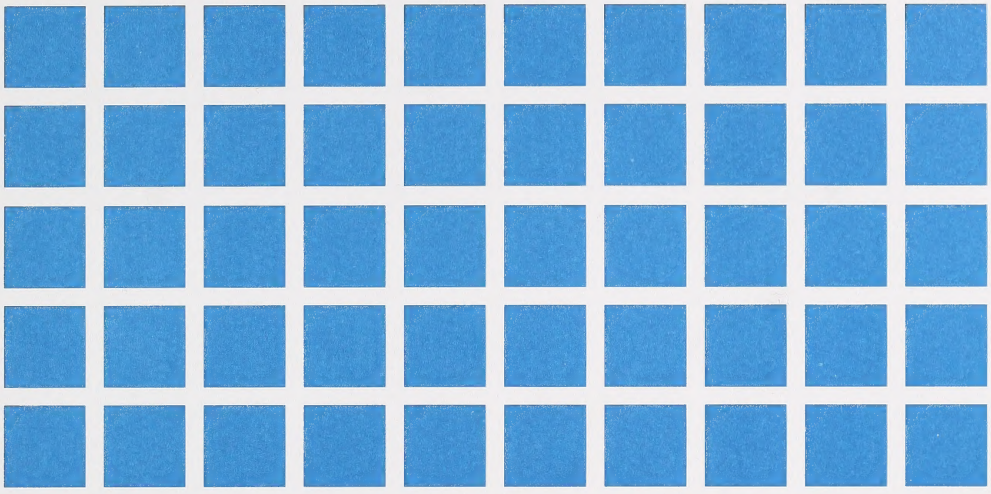


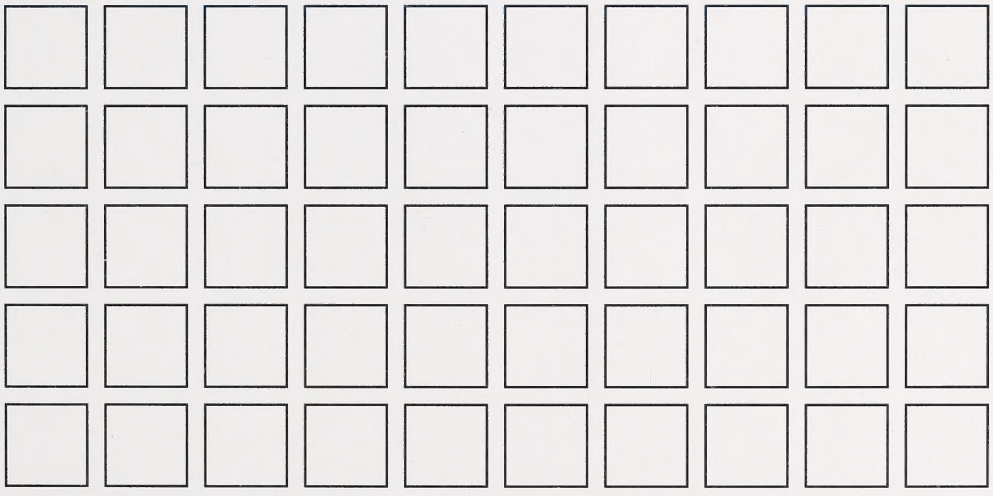
xy-Tiles

-*xy*-Tiles

1-Tiles



−1-Tiles



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